ESP
Kouba
Worksheet 6

1.) Evaluate the following definite integrals.

a.) $\int_0^1 (x+1)^3\,dx$ b.) $\int_0^1 (x+1)^{300}\,dx$

c.) $\int_0^1 x^4(x^5+1)^{300}\,dx$ d.) $\int_1^e \frac{1}{x}\,dx$

e.) $\int_0^{e^2-1} \frac{1}{x+1}\,dx$ f.) $\int_0^1 \frac{3x^2}{x^3+1}\,dx$

g.) $\int_0^1 \frac{\sec^2(x^2+1)\cdot 2x}{\tan(x^2+1)}\,dx$ h.) $\int_0^{\frac{\pi}{4}} \tan x\,dx$

2.) The temperature (° Fahrenheit) in a sealed room at time t (minutes) is given by

$$T(t) = \frac{200t}{4t^2+1}.$$

a.) Evaluate the temperature T at the following times: $t = 0, 1, 2, 10, 20$ min.

b.) What is the average temperature in the room during the time interval from $t = 0$ min. to $t = 20$ min.?

3.) Evaluate the following definite integrals <u>without</u> using calculus.

a.) $\int_0^2 3x\,dx$ b.) $\int_{-1}^2 3x\,dx$

c.) $\int_{-3}^3 \sqrt{9-x^2}\,dx$ d.) $\int_0^{3/2} \left(\sqrt{9-x^2} - f(x)\right) dx$,

where $f(x) = \begin{cases} 0, & x \le 0 \\ \sqrt{3}x, & x > 0 \end{cases}$.

e.) $\int_{-1}^1 x^{1/3}\,dx$ f.) $\int_{-1}^1 (x+x^3)^{1/3}\,dx$

4.) Evaluate the following limit:

$$\lim_{\Delta x \to 0} \frac{1}{\Delta x}\left\{\int_0^{5+\Delta x} \cos^5 4t\,dt - \int_0^5 \cos^5 4t\,dt\right\}$$

5.) Consider the function $f(x) = \dfrac{x}{x^4+3}$.

a.) Determine the maximum and minimum values of f on the interval $[-1,2]$.

b.) Use part a.) to show that

$$-\frac{3}{4} \le \int_{-1}^2 \frac{x}{x^4+3}\,dx \le \frac{3}{4}.$$

6.) Assume that f' is continuous for x in $[1,3]$ and that $f(1) = -1$ and $f(3) = 2$. Evaluate

$$\int_1^3 [x \cdot f'(x) + f(x)]\,dx.$$