ESP
Kouba
Worksheet 6 Solutions

1.) a.) $\int_0^1 (x+1)^3\,dx = \frac{1}{4}(x+1)^4\Big|_0^1 = \frac{15}{4}$

b.) $\int_0^1 (x+1)^{300}\,dx = \frac{1}{301}(x+1)^{301}\Big|_0^1 = \frac{2^{301}}{301} - \frac{1}{301}$

c.) $\int_0^1 x^4(x^5+1)^{300}\,dx = \frac{1}{5}\cdot\frac{1}{301}(x^5+1)^{301}\Big|_0^1 = \frac{2^{301}}{1505} - \frac{1}{1505}$

d.) $\int_1^e \frac{1}{x}\,dx = \ln x\Big|_1^e = \ln e - \ln 1 = 1$

e.) $\int_0^{e^2-1} \frac{1}{x+1}\,dx = \ln(x+1)\Big|_0^{e^2-1} = \ln e^2 - \ln 1 = 2$

f.) $\int_0^1 \frac{3x^2}{x^3+1}\,dx = \ln(x^3+1)\Big|_0^1 = \ln 2 - \ln 1 = \ln 2$

g.) $\int_0^1 \frac{\sec^2(x^2+1)\cdot 2x}{\tan(x^2+1)}\,dx = \ln(\tan(x^2+1))\Big|_0^1$

$= \ln(\tan 2) - \ln(\tan 1)$

h.) $\int_0^{\frac{\pi}{4}} \tan x\,dx = \int_0^{\frac{\pi}{4}} \frac{\sin x}{\cos x}\,dx = -\ln(\cos x)\Big|_0^{\frac{\pi}{4}}$

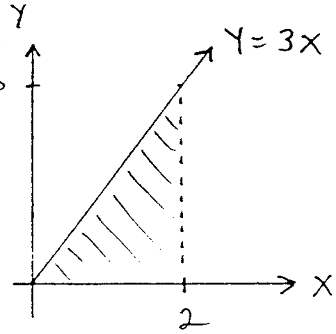
$= -\ln\left(\frac{\sqrt{2}}{2}\right) - -\ln(1) = \ln(\sqrt{2})$

2.) a.) $T(0) = 0^\circ$, $T(1) = 40^\circ$, $T(2) = 23.5^\circ$,
$T(10) = 4.9^\circ$, $T(20) = 2.5^\circ$

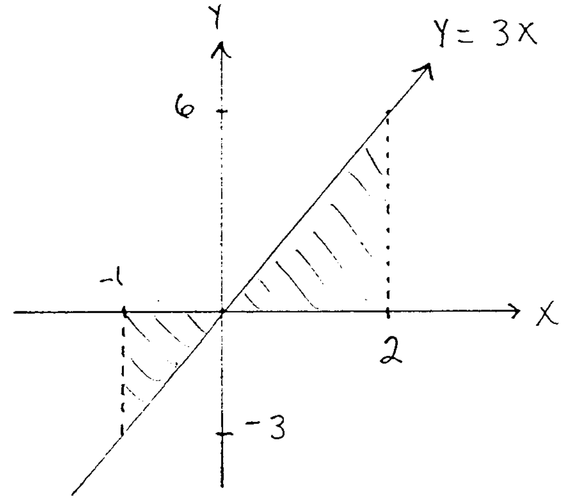
b.) $AVE = \frac{1}{20-0}\int_0^{20} \frac{200t}{4t^2+1}\,dt = \frac{1}{20}\int_0^{20} 25\cdot\frac{(8t)}{4t^2+1}\,dt$

$= \frac{25}{20}\cdot\ln(4t^2+1)\Big|_0^{20} = 9.22^\circ$

3.) a.)



$$\int_0^2 3x \, dx = \frac{1}{2}(2)(6) = 6.$$

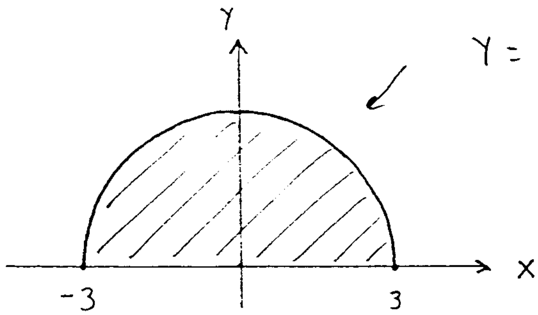


b.) $\int_{-1}^{2} 3x \, dx$

$$= \frac{1}{2}(2)(6) - \frac{1}{2}(1)(3)$$

$$= 6 - \frac{3}{2} = \frac{9}{2}$$

c.)

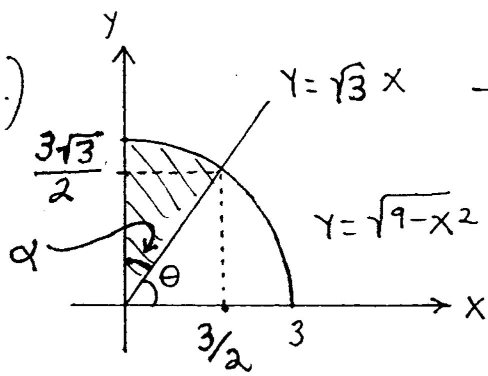


$Y = \sqrt{9-x^2} \Rightarrow x^2 + y^2 = 3^2$

(a circle of radius 3)

$$\int_{-3}^{3} \sqrt{9-x^2}\, dx = \frac{1}{2}\pi(3)^2 = \frac{9}{2}\pi$$

d.)



$\sqrt{3}\,x = \sqrt{9-x^2} \Rightarrow 3x^2 = 9 - x^2 \Rightarrow$

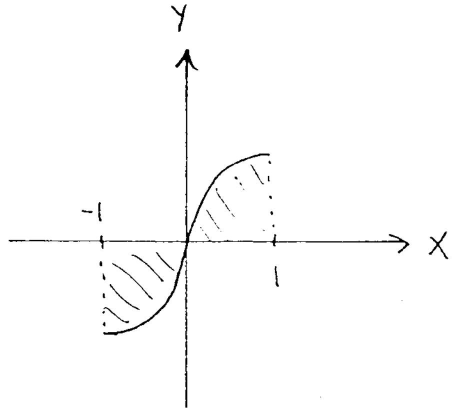
$4x^2 = 9 \Rightarrow x = 3/2$

$$\tan\theta = \frac{\frac{3\sqrt{3}}{2}}{3/2} = \sqrt{3} \quad \text{so}$$

$\theta = \frac{\pi}{3}$ rad. $= 60°$, $\alpha = \frac{\pi}{6}$ rad. $= 30°$, and

$$\int_0^{\sqrt{3}} \left(\sqrt{9-x^2} - f(x)\right) dx = \frac{\pi/6}{2\pi} \cdot \pi(3)^2 = \frac{3}{4}\pi.$$

e.)



By symmetry of odd function

$$\int_{-1}^{1} x^{1/3}\,dx = 0 .$$

f.) If $f(x) = (x+x^3)^{1/3}$, then

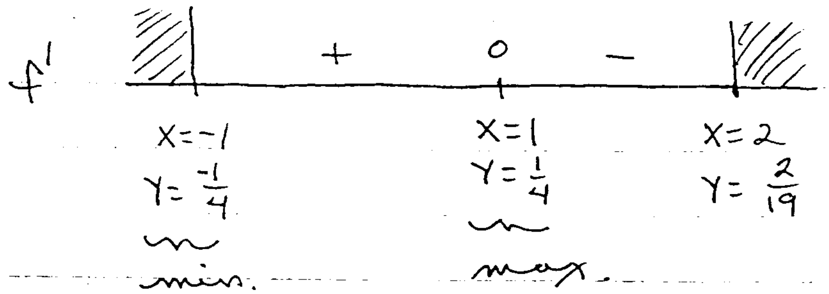
$f(-x) = (-x-x^3)^{1/3} = -(x+x^3)^{1/3} = -f(x)$, so that f

is an odd function. Thus, $\int_{-1}^{1} (x+x^3)^{1/3}\,dx = 0$.

4.) Let $F(x) = \int_0^x \cos^5 4t\,dt$, then $F'(x) = \cos^5 4x$ and

$$\lim_{\Delta x \to 0} \frac{F(5+\Delta x) - F(5)}{\Delta x} = F'(5) = \cos^5 20 .$$

5.) a.) $f(x) = \dfrac{x}{x^4+3} \Rightarrow f'(x) = \dfrac{3-3x^4}{(x^4+3)^2} = \dfrac{3(1-x)(1+x)(1+x^2)}{(x^4+3)^2} = 0$



$m = \frac{-1}{4}$

$M = \frac{1}{4}$

b.) If $\frac{-1}{4} \le \frac{x}{x^4+3} \le \frac{1}{4}$, then

$\frac{-1}{4}(2-(-1)) \le \int_{-1}^{2} \frac{x}{x^4+3}\,dx \le \frac{1}{4}(2-(-1))$ or $\frac{-3}{4} \le \int_{-1}^{2} \frac{x}{x^4+3}\,dx \le \frac{3}{4}$.

6.) $\int_1^3 [x f'(x) + f(x)]\,dx = x f(x) \Big|_1^3$

$= 3 \cdot f(3) - 1 \cdot f(1) = 3(2) - (-1) = 7$