ESP
Kouba
Worksheet 8 Solutions

1.) $\frac{dP}{dt} = kP \Rightarrow P = ce^{kt}$ and $t=0$, $P = \$750 \Rightarrow$

$P = 750e^{kt}$ and $t=1$, $P = \$840 \Rightarrow$

$840 = 750e^{k} \Rightarrow k = \ln 1.12 = .113328685 \Rightarrow$

$\boxed{P = 750\, e^{.113328685\, t}}$, so if $t = 20$ years,

$P = \$7234.72$.

2.)

# of years	amt. of \$
1	$750 + (.12)750 = (1.12)750$
2	$(1.12)750 + (.12)(1.12)750 = (1.12)^2 750$
3	$(1.12)^2 750 + (.12)(1.12)^2 750 = (1.12)^3 750$
⋮	⋮
t	$(1.12)^t 750$ or $\boxed{P = 750(1.12)^t}$

so if $t = 20$ years, $P = \$7234.72$.

3.) a.) $f'(x) = \dfrac{(1+\sqrt{x})\cdot \frac{1}{2\sqrt{x}} - \sqrt{x}\cdot\frac{1}{2\sqrt{x}}}{(1+\sqrt{x})^2} = \dfrac{1}{2\sqrt{x}(1+\sqrt{x})^2}$ so

$f(1.01) - f(1) \approx f'(1)\cdot(dx) = \frac{1}{8}(.01) = .00125$

b.) $g'(x) = 2x\cos(x^2)$ so

$g(\sqrt{\pi} - .1) - g(\sqrt{\pi}) \approx g'(\sqrt{\pi})\cdot(dx) = 2\sqrt{\pi}(-1)\cdot(-.1) = \frac{\sqrt{\pi}}{5} \approx .3545$.

c.) $h'(x) = \dfrac{1-2x}{3x^{2/3}(1+x)^2}$, $x: 8 \rightarrow 7.99$ so

$h(7.99) - h(8) \approx h'(8)\cdot(dx) = \frac{-5}{324}\cdot(-.01) = .00015432$.

d.) $f(x) = x^{1/5}$, $x: 32 \to 33$ ~~so~~

$(33)^{1/5} - (32)^{1/5} \approx f'(32)\cdot(dx) = \frac{1}{5}(32)^{-4/5}\cdot(1) = .0125$ and

$(33)^{1/5} \approx (32)^{1/5} + .0125 = 2.0125$.

4.) a.) $G'(x) = \arctan x \cdot 4x^3$

b.) i.) $G(1.01) - G(1) \approx G'(1)\cdot(dx) = 4\cdot\frac{\pi}{4}\cdot(.01) \approx .0314$.

ii.) $G(1.98) - G(2) \approx G'(2)\cdot(dx) = 32\arctan 2\cdot(-.02)$

$= -.70858$.

iii.) $G(\sqrt{3}+.01) - G(\sqrt{3}) \approx G'(\sqrt{3})\cdot(dx)$

$= \frac{\pi}{3}\cdot 4(3)^{3/2}(.01) \approx .21766$.

5.) a.) $\int e^x\,dx = e^x + c$

b.) $\int e^{-x}\,dx = -e^{-x} + c$

c.) $\int \frac{e^x}{1+e^x}\,dx = \ln(1+e^x) + c$

d.) $\int (1+e^{-x})\,dx = x - e^{-x} + c$

e.) $\int \frac{1}{1+e^x}\,dx = \int \frac{(1+e^x) - e^x}{1+e^x}\,dx$

$= \int \left[1 - \frac{e^x}{1+e^x}\right]dx = x - \ln(1+e^x) + c$

f.) $\int x e^x \, dx$ (let $u = x$, $dv = e^x \, dx$
$du = dx$, $v = e^x$)

$= x e^x - \int e^x \, dx = x e^x - e^x + c$

g.) $\int x^2 e^x \, dx$ (let $u = x^2$, $dv = e^x \, dx$
$du = 2x \, dx$, $v = e^x$)

$= x^2 e^x - 2\left(\int x e^x \, dx\right) = x^2 e^x - 2\left[x e^x - e^x + c\right]$

h.) $\int x e^{x^2} \, dx = \frac{1}{2} e^{x^2} + c$

i.) $\int x^2 e^{x^2} \, dx$ can't be integrated!

j.) $\int x^2 e^{x^3} \, dx = \frac{1}{3} e^{x^3} + c$

k.) $\int x^{\frac{1}{2}} \, dx = \frac{2}{3} x^{3/2} + c$

l.) $\int x^{-1/2} \, dx = 2 x^{1/2} + c$

m.) $\int \frac{\sin \sqrt{x}}{\sqrt{x}} \, dx = -2 \cos \sqrt{x} + c$

n.) $\int (1-x)^{1/2} \, dx = -\frac{2}{3} (1-x)^{3/2} + c$

o.) $\int x \sqrt{1-x} \, dx$ (let $u = 1-x$, $x = 1-u$,
$du = -dx$, $-du = dx$)

$= \int (1-u)\sqrt{u} \cdot -du = -\int (u^{1/2} - u^{3/2}) \, du$

$= -\left(\frac{2}{3} u^{3/2} - \frac{2}{5} u^{5/2}\right) + c = -\frac{2}{3}(1-x)^{3/2} + \frac{2}{5}(1-x)^{5/2} + c$

p.) $\int x(1-x^2)^{\frac{1}{2}}\,dx = -\frac{1}{3}\cdot(1-x^2)^{\frac{3}{2}} + c$

q.) $\int (x^{-\frac{1}{2}}+1)\,dx = 2x^{\frac{1}{2}} + x + c$

r.) $\int \frac{(1+\sqrt{x})^{10}}{\sqrt{x}}\,dx = \frac{2}{11}\cdot(1+\sqrt{x})^{11} + c$

s.) $\int (x-1)^{-\frac{1}{2}}\,dx = 2(x-1)^{\frac{1}{2}} + c$

t.) $\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + c$

u.) $\int \frac{1}{|x|\sqrt{x^2-1}}\,dx = \operatorname{arcsec} x + c$

v.) $\int x(1-x^2)^{-\frac{1}{2}}\,dx = -(1-x^2)^{\frac{1}{2}} + c$

w.) $\int \sin\theta\,d\theta = -\cos\theta + c$

x.) $\int \sin^4\theta\cdot\cos\theta\,d\theta = \frac{1}{5}\sin^5\theta + c$

y.) $\int (1+2\tan\theta+\tan^2\theta)\,d\theta = \int(\sec^2\theta + 2\tan\theta)\,d\theta$

$= \tan\theta + 2\ln|\sec x| + c$

z.) $\int \frac{\sec^2(\ln(e^{1+\sqrt{x}}))}{2\sqrt{x}}\,dx$ (Let $u = \ln(e^{1+\sqrt{x}})$,

$du = \frac{1}{\cancel{e^{1+\sqrt{x}}}}\cdot\cancel{e^{1+\sqrt{x}}}\cdot\frac{1}{2\sqrt{x}}\,dx$

$= \frac{1}{2\sqrt{x}}\,dx$)

$= \int \sec^2 u\,du$

$= \tan u + c$

$= \tan(\ln(e^{1+\sqrt{x}})) + c$.

6.) $\int_0^1 x f''(x)\,dx$ $\quad$ (let $u = x$, $dv = f''(x)\,dx$
$du = dx$, $v = f'(x)$)

$= x f'(x)\Big|_0^1 - \int_0^1 f'(x)\,dx$

$= f'(1) - f(x)\Big|_0^1 = 3 - (f(1) - f(0)) = 3 - 0 = 3$.

7.) a.) $\frac{dY}{dx} = x\,Y^4 \rightarrow \int \frac{1}{Y^4}\,dy = \int x\,dx \rightarrow$

$\frac{Y^{-3}}{-3} = \frac{x^2}{2} + c \rightarrow Y^{-3} = -\frac{3}{2}x^2 + c \Rightarrow$

$Y = \left(-\frac{3}{2}x^2 + c\right)^{-1/3}$ and $Y = 0$.

b.) $f(x) = 3\int_0^x f(t)\,dt \rightarrow f'(x) = 3 f(x) \rightarrow$

$f(x) = c e^{3x}$, and $f(0) = 3\int_0^0 f(t)\,dt$
$= 3(0) = 0$,

so $0 = Ce^{3(0)} = C(1) = C$ and

$f(x) = 0$.