ESP
Kouba
Worksheet 10

1.) Determine whether the following series converge or diverge. State the name of the test you use.

a.) $\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^{-8n}$ b.) $\sum_{n=2}^{\infty} \frac{n+30}{n^2\sqrt{n}}$

c.) $\sum_{n=0}^{\infty} \frac{1}{\sqrt{n+1}+\sqrt{n}}$ d.) $\sum_{n=0}^{\infty} (-1)^{n+2} \cdot \frac{10^{2n}}{(3n)!}$

e.) $\sum_{n=0}^{\infty} \left(\frac{-1}{10}\right)^n \cdot \frac{n^2+1}{n^2+7}$ f.) $\sum_{n=1}^{\infty} (-1)^n \frac{\left(1+\frac{1}{n}\right)\sqrt{n}}{4^n}$

g.) $\sum_{n=1}^{\infty} \frac{\cos n}{n^2}$ h.) $\sum_{n=1}^{\infty} n \sin\frac{1}{n}$

i.) $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{n^2-1}}$ j.) $\sum_{n=3}^{\infty} \left(\frac{1}{\ln n} - \frac{1}{\ln(n+1)}\right)$

k.) $\sum_{n=0}^{\infty} \frac{\arctan n}{1+n^2}$

2.) Determine the interval of convergence for each of the following power series.

a.) $\sum_{n=1}^{\infty} \frac{3^n x^n}{n^2}$ b.) $\sum_{n=1}^{\infty} \frac{(2x)^{n+1}}{n!}$

c.) $\sum_{n=0}^{\infty} (x-5)^n$ d.) $\sum_{n=0}^{\infty} 2^{n+1}(7-3x)^{2n}$

e.) $\sum_{n=0}^{\infty} (n+1)!\, x^{2n+1}$ f.) $\sum_{n=2}^{\infty} \ln n \cdot (2-x)^n$

g.) $\sum_{n=1}^{\infty} (n^2 x)^n$

h.) $\sum_{n=2}^{\infty} \frac{n^2+1}{n^3-1} (9)^n (x-3)^{2n}$

i.) $\sum_{n=1}^{\infty} \frac{(n!)^2}{n^{2n}} x^n$

3.) The series $\sum_{n=1}^{\infty} a_n x^n$ converges for $x=2$ and diverges for $x=-5$.

a.) For what other values of x must the series converge?

b.) For what other values of x must the series diverge?

4.) The series $\sum_{n=1}^{\infty} a_n (x+2)^n$ converges for $x=-3$ and diverges for $x=10$.

a.) For what other values of x must the series converge?

b.) For what other values of x must the series diverge?

5.) Let $\{a_n\}$ and $\{b_n\}$ be sequences of positive real numbers. Assume that both of the series $\sum_{n=1}^{\infty} a_n^2$ and $\sum_{n=1}^{\infty} b_n^2$ converge.

a.) Show that $\sum_{n=1}^{\infty} (3a_n^2 - 2b_n^2)$ converges.

b.) Show that $\sum_{n=1}^{\infty} a_n b_n$ converges.

c.) Show that $\sum_{n=1}^{\infty} a_n \cdot \frac{1}{n}$ converges.

6.) Use the power series for e^x, $\sin x$, and $\cos x$ to derive the first five nonzero terms of the power series for each of the following.

a.) $e^x + \sin x$ b.) $x \cdot \cos\sqrt{x}$

c.) $e^{-x} \cdot \cos 2x$ d.) $\dfrac{\sin x^2}{e^x}$

7.) Use the power series $\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \cdots$ to derive the first five nonzero terms of the power series for each of the following.

a.) $\dfrac{1}{1+x}$ b.) $\ln(1+x)$

c.) $\dfrac{1}{1+x^2}$ d.) $\arctan x$

e.) $\dfrac{2x}{1+x^2}$ f.) $\ln(1+x^2)$

g.) $\dfrac{1}{2+x}$

8.) Use power series to evaluate the following limits.

a.) $\displaystyle\lim_{x\to 0} \frac{(e^{x^2}-1)^2}{1-\frac{x^2}{2}-\cos x}$ b.) $\displaystyle\lim_{x\to 0} \frac{(1-\cos x^2)^3}{(x-\sin x)^4}$