ESP
Kouba
Worksheet 11

1.) Use the Taylor polynomial $P_3(x;0)$ to estimate each of the following. Use a calculator to compare your estimate to the exact value.

a.) $e^{0.1}$ b.) e^4

c.) $\sin(0.2)$ d.) $\ln(1.5)$

2.) Compute the first four nonzero terms of the Taylor series about $a=1$ for each of the following.

a.) e^x b.) $\sqrt{x}$

3.) a.) Use the first six terms of the Maclaurin series for e^{-x} to estimate e^{-1}.

b.) Estimate the error in part a.) using

i.) $|R_n| < a_{n+1}$ for alternating series

ii.) $|R_n(x;a)| = \left| \dfrac{f^{(n+1)}(c)}{(n+1)!}(x-a)^{n+1} \right|$, the Taylor remainder

iii.) a calculator

4.) Expand $f(x) = 3x^4 - x^3 + 2x^2 - x + 5$ in powers of $(x-1)$.

5.) For each of the following, show that the Taylor remainder $R_n(x;0) \to 0$ as $n \to \infty$ for the indicated values of x.

a.) e^x for all values of x

b.) $\ln(1+x)$ for $\frac{-1}{2} < x < 1$

6.) Consider the integral $\int_0^1 \cos\sqrt{x}\, dx$.

a.) Using an appropriate Taylor series, estimate the exact value of the integral with an accuracy of 0.00001.

b.) Evaluate the integral using the Fundamental Theorem of Calculus.