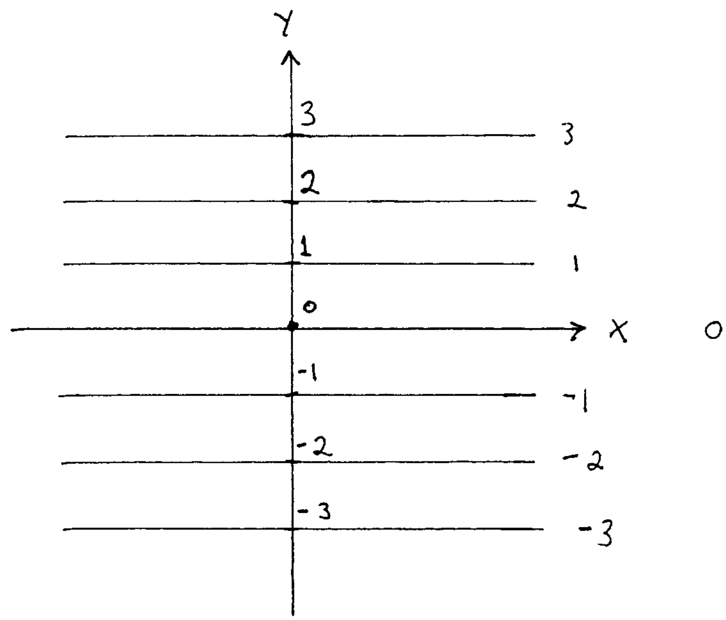
ESP
Kouba
Worksheet 2 Solutions

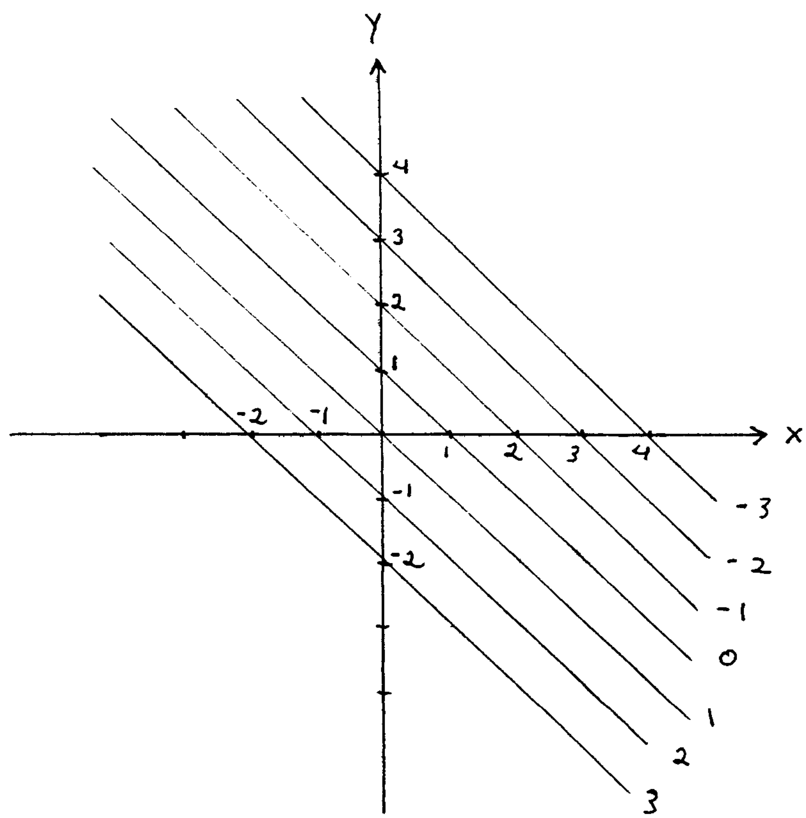
1.) a.) $z = y$

z	level curve
-3	$y = -3$
-2	$y = -2$
-1	$y = -1$
0	$y = 0$
1	$y = 1$
2	$y = 2$
3	$y = 3$



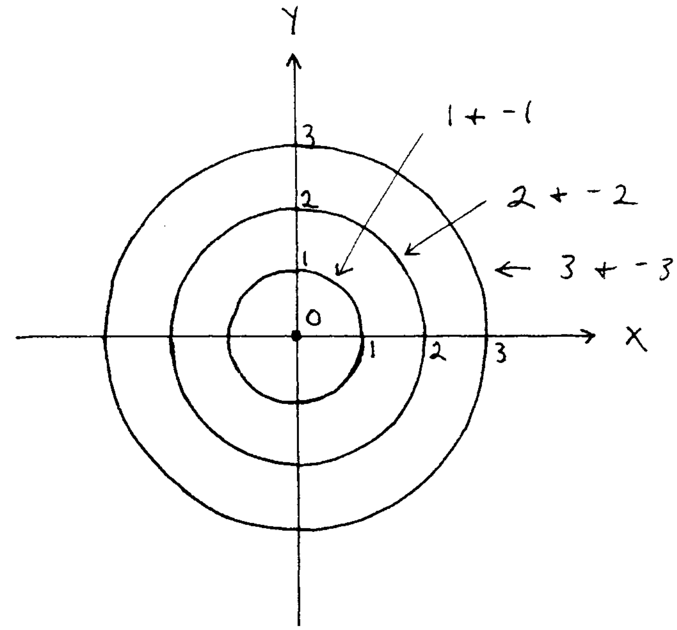
b.) $z = 1 - x - y$

z	level curve
-3	$y = 4 - x$
-2	$y = 3 - x$
-1	$y = 2 - x$
0	$y = 1 - x$
1	$y = -x$
2	$y = -1 - x$
3	$y = -2 - x$



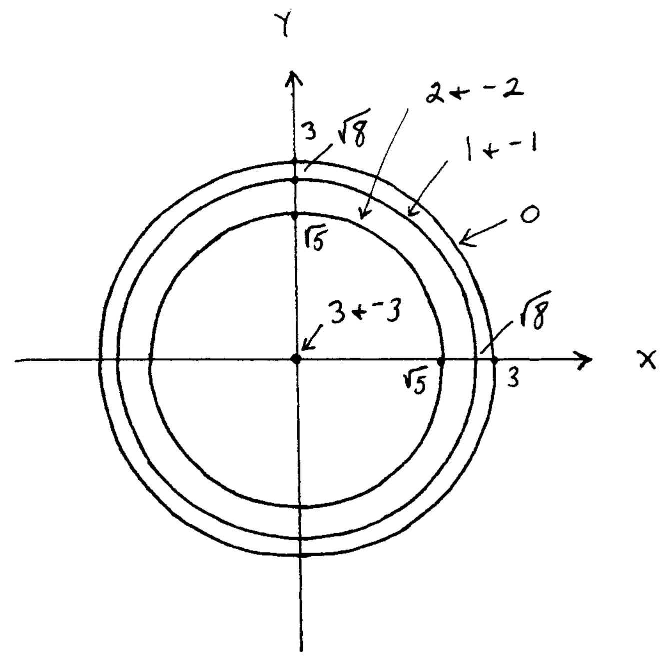
c.) $z^2 = x^2 + y^2$

z	level curve
-3	$9 = x^2 + y^2$
-2	$4 = x^2 + y^2$
-1	$1 = x^2 + y^2$
0	$0 = x^2 + y^2$
1	$1 = x^2 + y^2$
2	$4 = x^2 + y^2$
3	$9 = x^2 + y^2$



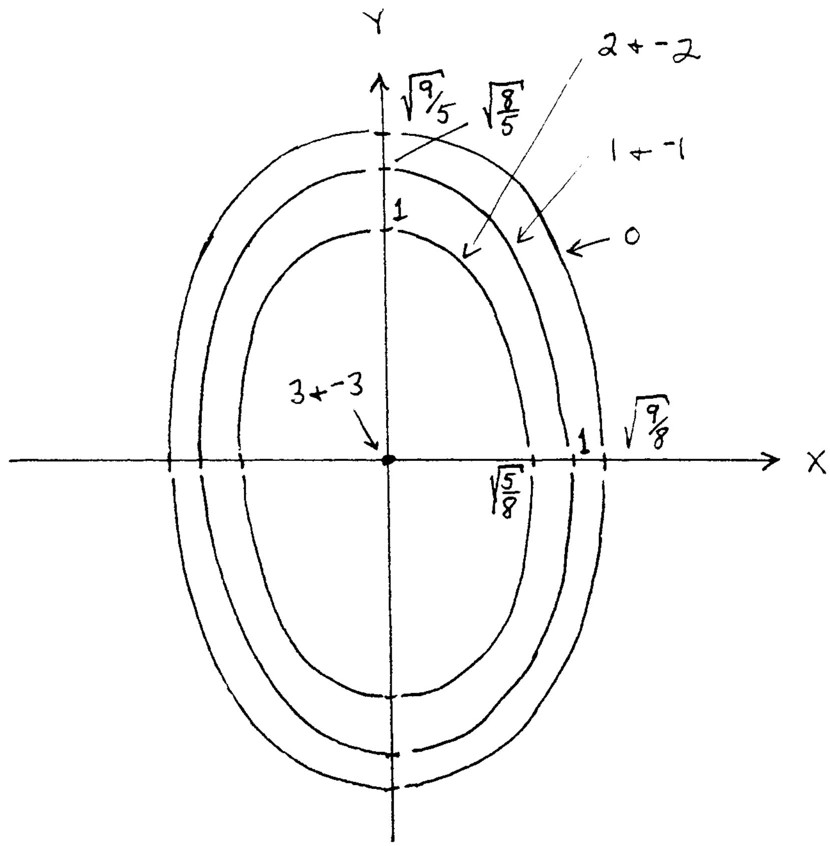
d.) $x^2 + y^2 + z^2 = 3^2$

z	level curve
-3	$x^2 + y^2 = 0$
-2	$x^2 + y^2 = 5$
-1	$x^2 + y^2 = 8$
0	$x^2 + y^2 = 9$
1	$x^2 + y^2 = 8$
2	$x^2 + y^2 = 5$
3	$x^2 + y^2 = 0$



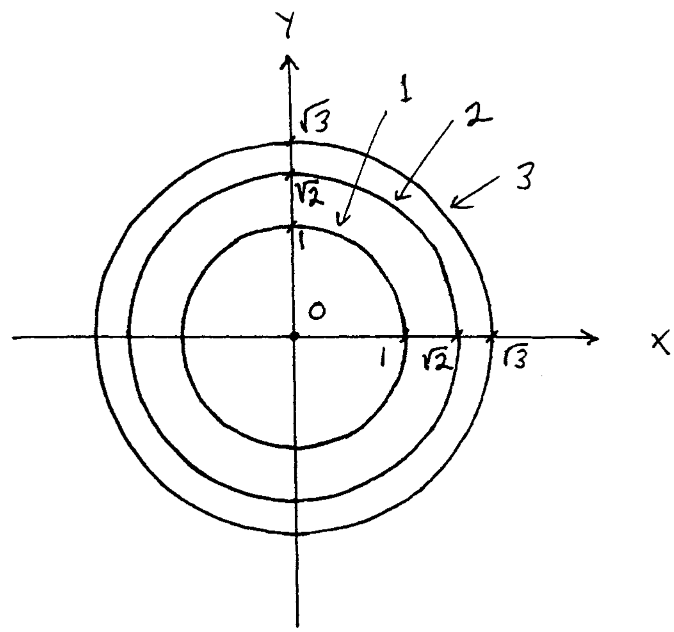
e.) $8x^2+5y^2+z^2=3^2$

z	level curve
-3	$8x^2+5y^2=0$
-2	$\frac{x^2}{\frac{5}{8}}+\frac{y^2}{1}=1$
-1	$\frac{x^2}{1}+\frac{y^2}{\frac{8}{5}}=1$
0	$\frac{x^2}{\frac{9}{8}}+\frac{y^2}{\frac{9}{5}}=1$
1	$\frac{x^2}{1}+\frac{y^2}{\frac{8}{5}}=1$
2	$\frac{x^2}{\frac{5}{8}}+\frac{y^2}{1}=1$
3	$8x^2+5y^2=0$



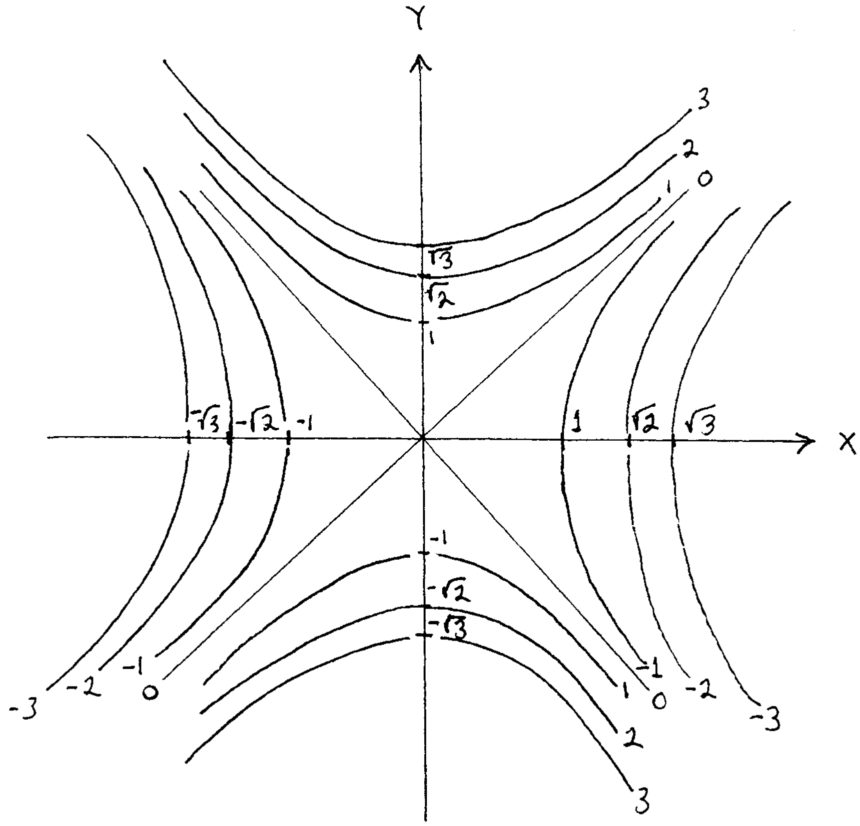
f.) $z=x^2+y^2$

z	level curve
-3	none
-2	none
-1	none
0	$x^2+y^2=0$
1	$x^2+y^2=1$
2	$x^2+y^2=2$
3	$x^2+y^2=3$



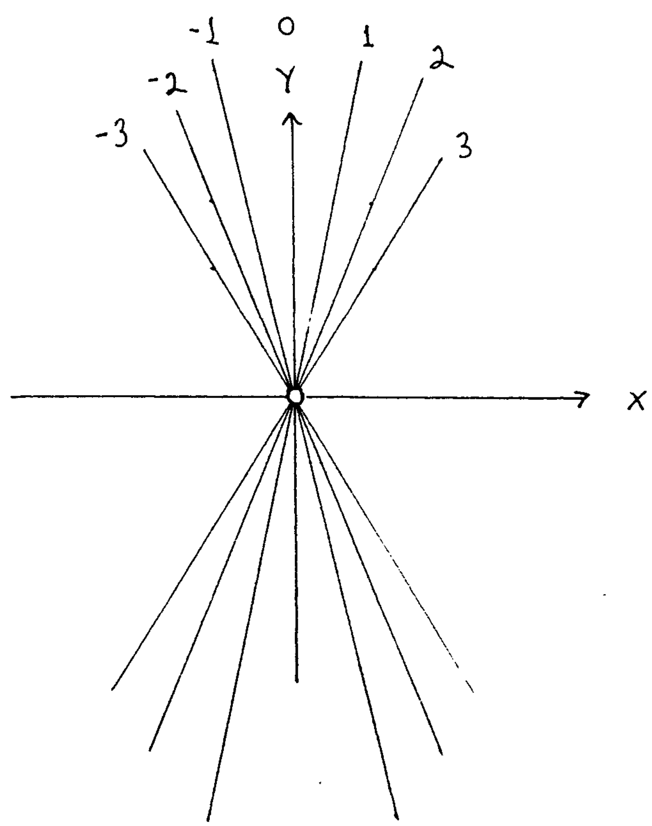
g.) $z = y^2 - x^2$

z	level curve
-3	$3 = x^2 - y^2$
-2	$2 = x^2 - y^2$
-1	$1 = x^2 - y^2$
0	$0 = x^2 - y^2$
1	$1 = y^2 - x^2$
2	$2 = y^2 - x^2$
3	$3 = y^2 - x^2$



h.) $z = \frac{6x}{y}$, $y \neq 0$

z	level curve
-3	$y = -2x$
-2	$y = -3x$
-1	$y = -6x$
0	$x = 0$
1	$y = 6x$
2	$y = 3x$
3	$y = 2x$



2.) a.) plane including the x-axis lying at 45° to y-axis and z-axis

b.) plane with intercepts x=1, y=1, z=1

c.) two cones with apices meeting at the origin

d.) sphere with radius 3 centered at the origin

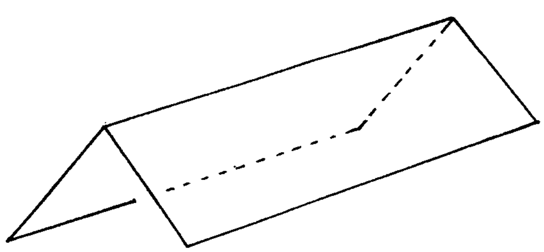
e.) ellipsoid centered at the origin with semi-axis lengths of $x = \sqrt{\frac{9}{8}}$, $y = \sqrt{\frac{9}{5}}$, and $z = 3$

f.) paraboloid with vertex at the origin

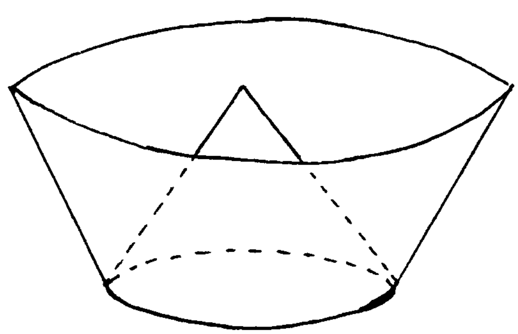
g.) hyperbolic paraboloid (saddle)

h.) helix (ribbon) with 180° twist

3.) a.)

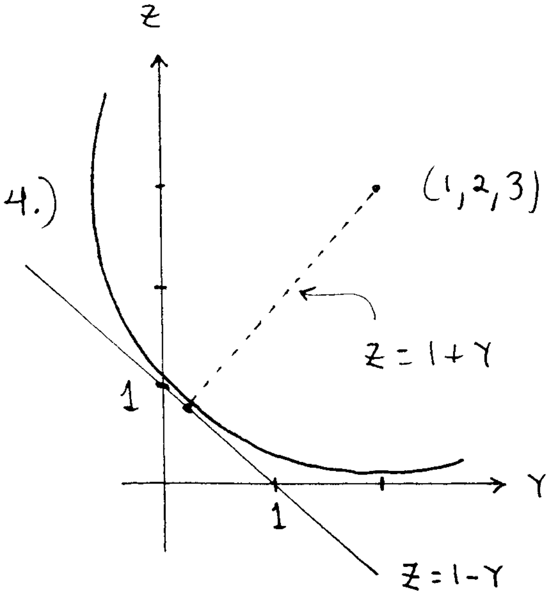


b.)



side view

4.)



find $\cap$ of lines:

$1+Y = 1-Y \rightarrow Y=0,\ z=1$

so distance between $(1,2,3)$ and $(1,0,1)$ is $\sqrt{8}$, i.e., $r=\sqrt{8}$

so sphere is $(x-1)^2+(Y-2)^2+(z-3)^2=8$.

5.) a.) $z_x = Y^2+\frac{1}{x}$, $z_Y = 2xY+e^Y$

b.) $z_x = xe^Y\cdot\frac{1}{1+x^2} + e^Y\cdot\arctan x$

$z_Y = xe^Y\arctan x$

c.) $z_x = \frac{1}{2}(x-Y^2)^{-1/2}\cdot(1)$, $z_Y = \frac{1}{2}(x-Y^2)^{-1/2}\cdot(-2Y)$

d.) $z_x = \frac{3x^2}{Y^2}+Y\cos(xY)$, $z_Y = \frac{-2x^3}{Y^3}+x\cos(xY)$

e.) $z_x = \frac{(x^2+Y^2)(1)-x(2x)}{(x^2+Y^2)^2}$, $z_Y = -2xY\,(x^2+Y^2)^{-2}$

f.) $z_x = 5[e^{x^2-Y}+\tan(3Y)]^4\cdot\{2x\cdot e^{x^2-Y}\}$

$z_Y = 5[e^{x^2-Y}+\tan(3Y)]^4\cdot\{-e^{x^2-Y}+3\cdot\sec^2(3Y)\}$

g.) $\ln z = (1+x^3)\cdot\ln Y$ so

$\frac{1}{z} \cdot z_x = 3x^2 \cdot \ln y$ or $z_x = y^{1+x^3} \cdot 3x^2 \cdot \ln y$ and

$z_y = (1+x^3) \cdot y^{x^3}$.

6.) $z = \ln(1+x^2+y^2) \rightarrow$

$z_x = \dfrac{2x}{1+x^2+y^2}$ and $z_y = \dfrac{2y}{1+x^2+y^2}$ and

$z_{xy} = \dfrac{-4xy}{(1+x^2+y^2)^2}$, then

$z_{xy} + z_x \cdot z_y = \dfrac{-4xy}{(1+x^2+y^2)^2} + \dfrac{4xy}{(1+x^2+y^2)^2} = 0$.

7.) a.) $z = x^2 + y^3 + y$

b.) $z = \frac{1}{2}x^2y^2 - xy$

c.) $z = x^4y^5 - x$

d.) impossible

e.) impossible

8.) a.) $z_x = y^2 - 3x^2$ at $(1,0,7)$ $m = -3$

b.) $z_y = 2xy$ at $(1,0,7)$ $m = 0$