ESP
Kouba
Worksheet 4

1.) Determine the limit as (x, y) approaches $(0, 0)$ for each of the following.

a.) $f(x, y) = \dfrac{x^4 - y^4}{x^2 + y^2}$

b.) $f(x, y) = \dfrac{2xy}{5x^4 + 3y^4}$

c.) $f(x, y) = \dfrac{x^{3/2} - xy}{x^{3/2} + y^3}$

2.) The two shortest sides of a right triangle are measured as 3 cm. and 4 cm., resp., with a maximum absolute error of 0.02 cm. for each measurement. Use differentials to approximate the maximum absolute error in measuring

a.) the hypotenuse.
b.) the area.

3.) Use differentials to approximate the change in $w = r^2 + 3sv + 2p^3$ if r changes from 1 to 1.02, s from 2 to 1.99, v from 4 to 4.01, and p form 3 to 2.97 .

4.) The dimensions of a rectangular room are $9 \times 12 \times 8$ ft. with possible errors of ± 0.01, ± 0.02, and ± 0.03 ft., resp. Calculate the length of the long diagonal across the room and the possible error in this measurement.

5.) A rectangular solid has sides of length 1.02, 3.01, and 4.2 cm.

a.) Compute the volume.
b.) Use a differential to estimate the volume.

6.) The specific gravity of an object is $s = A / (A - W)$, where A and W are the weights of the object in air and water, resp. If $A = 12$ lbs. and $W = 5$ lbs. with maximum absolute errors of 1/2 oz. in air and 1 oz. in water, what is the maximum absolute error in the calculated value of s ?

7.) Find dw/dt where $w = \ln(3u + v^2)$, $u = e^{-2t}$, and $v = t^3 - t^2$.

8.) Find $\partial w/\partial t$ and $\partial w/\partial s$ where $w = f(3t^2 - s)$ and $f'(x) = \sin x$.

9.) Find z_x where z satisfies $xy^2 + z^2 + \cos(xyz) = 4$.

10.) Assume that f is a differentiable function with $w = f(ax + by)$, where a and b are constants. Show that

$$a(\partial w/\partial y) = b(\partial w/\partial x).$$

11.) Assume that f is differentiable with $z = x\, f(xy)$. Show that

$$x \cdot z_x - y \cdot z_y = z.$$

12.) Assume that f and g are twice differentiable functions. Show that $u = f(x + at) + g(x - at)$ satisfies

$$a^2 \cdot \partial^2 u / \partial x^2 = \partial^2 u / \partial t^2,$$

where a is a constant.

13.) Find the critical points and classify each as a relative maximum, relative minimum, or saddle point.

a.) $f(x, y) = x^3 - 3xy^2 + 3y^2$

b.) $f(x, y) = 3x^2 - 6xy + y^2 + 12x - 16y + 1$

c.) $f(x, y) = x^2 - \ln(xy) + y^2$

14.) Find the shortest distance between the planes $2x + 3y - z = 2$ and $2x + 3y - z = 4$.

15.) Find the dimensions of the rectangular parallelepiped of maximum volume that can be inscribed inside the ellipsoid

$$16x^2 + 4y^2 + 9z^2 = 144 .$$

16.) Determine the minimum surface area of a closed rectangular box with volume 8 $ft.^3$

17.) Determine the maximum and minimum values of f on the given region.

a.) $f(x, y) = (x - 1)^2 + (y - 2)^2$ on the triangle with vertices (0, 0), (0, 4), and (5, 0)

b.) $f(x, y) = xy$ on the unit circle