ESP
Kouba
Worksheet 5

1.) Compute $\frac{\partial z}{\partial x}$ and $\frac{\partial^2 z}{\partial x^2}$ for each of the following.

a.) $z = e^{\tan(y-x)} \cdot \ln(x^2+y^3)$

b.) $z = g(u,v)$ with $u = 2x-3y$, $v = x^2+y^2$

2.) Assume that f is differentiable and $z = f\left(\frac{x}{y}\right)$. Show that

$$x \cdot z_x + y \cdot z_y = 0 .$$

3.) Classify the critical points for

$$f(x,y) = xy^2 - x^2y + x - y .$$

4.) Find all points on the surface $xyz = 8$ which are closest to the origin.

5.) Let R be the region bounded by the graphs of $y = \sqrt{x}$ and $y = \frac{1}{4}x$.

a.) Describe R using vertical cross-sections.
b.) Describe R using horizontal cross-sections.
c.) Set up iterated integrals for each of the following

i.) $\iint_R f(x,y)\, dx\, dy$ ii.) $\iint_R f(x,y)\, dy\, dx$.

6.) Let R be the region above the x-axis and below the semi-circle of radius 2 centered at $(2,0)$.

a.) Describe R using vertical cross-sections.

b.) Describe R using horizontal cross-sections.

c.) Describe R in the form

$$\alpha \le \theta \le \beta, \quad r_1(\theta) \le r \le r_2(\theta).$$

d.) Describe R in the form

$$a \le r \le b, \quad \theta_1(r) \le \theta \le \theta_2(r).$$

7.) Sketch each of the following regions.

a.) $0 \le x \le 1, \quad x \le y \le e^x$

b.) $-1 \le y \le \sqrt{3}, \quad \arctan y \le x \le \frac{\pi}{3}$

c.) $\frac{\pi}{4} \le \theta \le \frac{3\pi}{4}, \quad 3\csc\theta \le r \le 6\sin\theta$

d.) $0 \le r \le \sqrt{2}, \quad \frac{\pi}{6} \le \theta \le \frac{\pi}{4}$ and
$\sqrt{2} \le r \le 2, \quad \frac{\pi}{6} \le \theta \le \arcsin(1/r)$

8.) Compute the approximating sum
$\sum_{i=1}^{n} f(P_i)\cdot A_i$ for the function $f(x,y) = x^2 + y^2$
on the square region R with vertices

(1,0), (3,0), (1,2), and (3,2), which is divided into four equal squares using the geometric center of each R_i as the P_i for $i = 1, 2, 3, 4$.

9.) Evaluate each of the following.

a.) $\int_0^1 \int_2^3 2x^2 y \; dy \, dx$

b.) $\int_1^2 \int_1^x \frac{x^2}{y^2} \; dy \, dx$

c.) $\int_0^{\sqrt{\pi}} \int_0^y \sin y^2 \; dx \, dy$

d.) $\int_0^1 \int_y^1 \sqrt{1+x^2} \; dx \, dy$

e.) $\int_0^8 \int_{y^{1/3}}^2 e^{x^4} \; dx \, dy$

10.) Consider the solid tetrahedron with vertices (0,0,0), (1,0,0), (0,2,0), and (0,0,3).

a.) Its top surface lies in a plane. Determine an equation for this plane.

b.) Compute the volume of the tetrahedron.