ESP
Kouba
Worksheet 5

1.) Compute \( \frac{\partial z}{\partial x} \) and \( \frac{\partial^2 z}{\partial x^2} \) for each of the following.
   
a.) \( z = e^{\tan(y-x)} \cdot \ln(x^2+y^2) \)
   
b.) \( z = g(u,v) \) with \( u = 2x - 3y \), \( v = x^2 + y^2 \)

2.) Assume that \( f \) is differentiable and \( z = f\left(\frac{x}{y}\right) \).
   
   Show that
   
   \[ x \cdot z_x + y \cdot z_y = 0 \]

3.) Classify the critical points for
   
   \[ f(x,y) = xy^2 - x^2y + x - y \]

4.) Find all points on the surface \( xy + z = 8 \) which are closest to the origin.

5.) Let \( R \) be the region bounded by the graphs of \( y = \sqrt{x} \) and \( y = \frac{1}{4}x \).
   
   a.) Describe \( R \) using vertical cross-sections.
   
   b.) Describe \( R \) using horizontal cross-sections.
   
   c.) Set up iterated integrals for each of the following
   
   i.) \( \iint_R f(x,y) \, dx \, dy \)
   
   ii.) \( \iint_R f(x,y) \, dy \, dx \)
6.) Let $R$ be the region above the $x$-axis and below the semi-circle of radius 2 centered at $(2,0)$.

a.) Describe $R$ using vertical cross-sections.
b.) Describe $R$ using horizontal cross-sections.
c.) Describe $R$ in the form $\alpha \leq \theta \leq \beta$, $r_1(\theta) \leq r \leq r_2(\theta)$.
d.) Describe $R$ in the form $a \leq r \leq b$, $\phi_1(r) \leq \phi \leq \phi_2(r)$.

7.) Sketch each of the following regions.

a.) $0 \leq x \leq 1$, $x \leq y \leq e^x$
b.) $-1 \leq y \leq \sqrt{3}$, $\arctan y \leq x \leq \frac{\pi}{3}$
c.) $\frac{\pi}{4} \leq \theta \leq \frac{3\pi}{4}$, $3 \sec \theta \leq r \leq 6 \sin \theta$
d.) $0 \leq r \leq \sqrt{2}$, $\frac{\pi}{6} \leq \theta \leq \frac{\pi}{4}$ and $\sqrt{2} \leq r \leq 2$, $\frac{\pi}{6} \leq \theta \leq \arcsin (\frac{1}{r})$

8.) Compute the approximating sum $\sum_{i=1}^{n} f(P_i) \cdot A_i$ for the function $f(x, y) = x^2 + y^2$ on the square region $R$ with vertices
(1,0), (3,0), (1,2), and (3,2), which is divided into four equal squares using the geometric center of each \( R_i \) as the \( P_i \) for \( i = 1, 2, 3, 4 \).

9.) Evaluate each of the following:
   a.) \( \int_0^1 \int_2^3 2x^2 \gamma \ dy \ dx \)
   b.) \( \int_{1}^{2} \int_{1}^{x} \frac{x^2}{y^2} \ dy \ dx \)
   c.) \( \int_{0}^{\sqrt{\pi}} \int_{0}^{\gamma} \sin \gamma^2 \ dx \ dy \)
   d.) \( \int_{0}^{1} \int_{0}^{1} \sqrt{1 + x^2} \ dx \ dy \)
   e.) \( \int_{0}^{8} \int_{0}^{2} \ e^{x^4} \ dx \ dy \)

10.) Consider the solid tetrahedron with vertices \((0,0,0), (1,0,0), (0,2,0), \) and \((0,0,3)\).
   a.) Its top surface lies in a plane. Determine an equation for this plane.
   b.) Compute the volume of the tetrahedron.