ESP
Kouba

Worksheet 6

1.) Assume that \( w = f(x, y, z) \) and \( f(u - t, t, u) = 0 \).

Show that \( f_y + f_z = 0 \).

2.) Evaluate the following double integrals.

a.) \[ \int_{0}^{\frac{\pi}{4}} \int_{0}^{\cos \theta} 3r^2 \sec \theta \, dr \, d\theta \]

b.) \[ \int_{0}^{1} \int_{0}^{\sqrt[2]{1-x^2}} e^{x^2+y^2} \, dy \, dx \]

c.) \[ \int_{0}^{1} \int_{0}^{\frac{x}{\sqrt{3}}} \frac{\sqrt{x^2+y^2}}{\sqrt{3}} \, dy \, dx \]

3.) Assume that region \( R \) is described in polar coordinates by \( \alpha \leq \theta \leq \beta \) and \( 0 \leq r \leq f(\theta) \).

Show that the area of region \( R \) is

\[ \text{Area} = \frac{1}{2} \int_{\alpha}^{\beta} [f(\theta)]^2 \, d\theta. \]

4.) Consider the cylinder above the circle \( (x - \frac{1}{2})^2 + y^2 = \frac{1}{4} \) in the \( xy \)-plane and below the plane \( z = x + 1 \). Compute its volume.

5.) Determine an equation for the plane tangent to the surface \( z = x^2 + y^4 \) at the point \( (1, -1, 2) \).
6.) A thin lamina lies in the triangular region with vertices (0,0), (0,2), and (3,2). Density at point \((x,y)\) is \(f(x,y) = x^2 + y\).

Set up but do not evaluate the integrals which represent

a.) its centroid.

b.) its center of mass.

c.) the moment about
   i.) the line \(x=1\).
   ii.) the line \(y=2\).

d.) the moment of inertia about
   i.) the origin
   ii.) the line \(x = 4\).

7.) Draw the solids described below.

a.) \(-2 \leq x \leq 0, -\sqrt{4-x^2} \leq y \leq \sqrt{4-x^2}, 0 \leq z \leq \sqrt{x^2 + y^2}\)

b.) \(\frac{\pi}{4} \leq \theta \leq \frac{\pi}{2}, 0 \leq r \leq 2, r^2 \leq z \leq 4\)

8.) Use rectangular coordinates to describe the tetrahedron with corners \((2,0,2), (1,0,2), (1,1,2), \) and \((1,1,1)\).

a.) First project it onto the \(xy\) - plane.

b.) First project it onto the \(xz\) - plane.

9.) Let \(R\) be the solid prism with vertices \((0,0,0), (0,0,1), (0,2,0), (0,2,1), (3,0,0), \) and \((3,0,1)\). Evaluate \(\int_R 1 \, dv\). What does your answer represent?

10.) Compute \(\int_R z \, dv\), where \(R\) is the region above the rectangle whose vertices are \((0,0,0), (2.0,0), (2.3,0), \) and \((0,3,0)\) and below the plane \(z = x + 2y\).