ESP

Kouba

Worksheet 6

1.) Assume that $w = f(x,y,z)$ and $f(u-t, t, u) = 0$.

Show that $f_y + f_z = 0$.

2.) Evaluate the following double integrals.

a.) $\int_0^{\frac{\pi}{4}} \int_0^{\cos\theta} 3r^2 \sec\theta \, dr \, d\theta$

b.) $\int_0^1 \int_0^{\sqrt{1-x^2}} e^{x^2+y^2} \, dy \, dx$

c). $\int_0^1 \int_{\frac{x}{\sqrt{3}}}^{x} \sqrt{x^2+y^2} \, dy \, dx$

3.) Assume that region R is described in polar coordinates by $\alpha \leq \theta \leq \beta$ and $0 \leq r \leq f(\theta)$. Show that the area of region R is

$$\text{Area} = \frac{1}{2} \int_\alpha^\beta \left[f(\theta)\right]^2 d\theta.$$

4.) Consider the cylinder above the circle $(x - \frac{1}{2})^2 + y^2 = \frac{1}{4}$ in the xy – plane and below the plane $z = x + 1$. Compute its volume.

5.) Determine an equation for the plane tangent to the surface $z = x^2 + y^4$ at the point $(1, -1, 2)$.

6.) A thin lamina lies in the triangular region with vertices (0,0), (0,2), and (3,2). Density at point (x,y) is $f(x,y) = x^2 + y$.

Set up but do not evaluate the integrals which represent

a.) its centroid.

b.) its center of mass.

c.) the moment about

i.) the line $x=1$.

ii.) the line $y=2$.

d.) the moment of inertia about

i.) the origin

ii.) the line $x = 4$.

7.) Draw the solids described below.

a.) $-2 \le x \le 0,\ -\sqrt{4-x^2} \le y \le \sqrt{4-x^2},\ 0 \le z \le \sqrt{x^2 + y^2}$

b.) $\frac{\pi}{4} \le \theta \le \frac{\pi}{2},\ 0 \le r \le 2,\ r^2 \le z \le 4$

8.) Use rectangular coordinates to describe the tetrahedron with corners (2,0,2), (1,0,2), (1,1,2), and (1,1,1).

a.) First project it onto the xy – plane.

b.) First project it onto the xz – plane.

9.) Let R be the solid prism with vertices (0,0,0), (0,0,1), (0,2,0), (0,2,1), (3,0,0), and (3,0,1). Evaluate $\int_R 1\, dv$. What does your answer represent?

10.) Compute $\int_R z\, dv$, where R is the region above the rectangle whose vertices are (0,0,0), (2,0,0), (2,3,0), and (0,3,0) and below the plane $z = x + 2y$.