

ESP

Kouba

Worksheet 7

1.) Describe each of the solid regions using rectangular, cylindrical, and spherical coordinates, resp.

- a.) the right circular cylinder of height 10 and with circular base of radius 3 in the xy -plane centered at the origin.
- b.) the ball inside the sphere $x^2 + y^2 + z^2 = 16$.
- c.) the cone above the surface $z = \sqrt{x^2 + y^2}$ and below the plane $z = 5$.
- d.) the region below the top half of $x^2 + y^2 + z^2 = 25$ and above the bottom half of $x^2 + y^2 + (z-5)^2 = 25$
- e.) the region enclosed by the surfaces $z = x^2 + y^2$ and $z = 9$.
- f.) the region enclosed by the surfaces $z = x^2 + y^2$ and $z = 2y + 4$.

2.) Sketch the solid regions described below.

- a.) $-3 \leq x \leq 3$, $-\sqrt{9-x^2} \leq y \leq 0$, $-2 \leq z \leq y^2$
- b.) $-\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4}$, $0 \leq r \leq 2 \cos \theta$, $0 \leq z \leq 3$
- c.) $\frac{\pi}{4} \leq \theta \leq \frac{5\pi}{4}$, $0 \leq \phi \leq \frac{\pi}{3}$, $\sec \phi \leq \rho \leq 2$

3.) Compute the average value of the function $f(x, y) = x + y$ over the region R described by $1 \leq x \leq 3$, $2 \leq y \leq 2x$.

4.) Describe each region in the two forms
i.) $\alpha \leq \theta \leq \beta$, $r_1(\theta) \leq r \leq r_2(\theta)$
ii.) $a \leq r \leq b$, $\theta_1(r) \leq \theta \leq \theta_2(r)$

a.) the region inside the curve $r = 2 \cos \theta$

b.) the region inside the curve $r = 1 - \cos \theta$ and outside the curve $r = 1$

5.) Evaluate the following integrals.

a.) $\int_1^2 \int_1^x \frac{x^2}{y^2} dy dx$

b.) $\int_0^1 \int_y^1 \sqrt{1+x^2} dx dy$

c.) $\int_2^3 \int_{-\sqrt{18-2y^2}}^{+\sqrt{18-2y^2}} 2x dx dy$

d.) $\int_1^2 \int_0^{\sqrt{\ln \theta}} \int_0^{2r\theta} \frac{e^{r^2}}{\theta^2 + 4} dz dr d\theta$

6.) Compute the volume of the smaller wedge cut from a sphere of radius 4 by two planes which meet at a diameter at an angle of $\frac{\pi}{3}$.

7.) Compute the volume of the solid bounded by the plane $z = x$ and the paraboloid $z = x^2 + y^2$.

8.) Let R be the region bounded by the paraboloids $z = x^2 + y^2$ and $z = 1 - x^2 - y^2$. Describe region R using

- a.) rectangular coordinates
- b.) cylindrical coordinates
- c.) spherical coordinates

9.) Let R be the region bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 4 + 2x + 4y$. Assume that density at point $P = (x, y, z)$ is given by $\delta(P) = \sqrt{x^2 + y^2}$. Compute the total mass of R . Set up but do not evaluate the integral.

10.) Let R be the region bounded by the surfaces $y = 0$ and $y = \sqrt{1 - x^2 - z^2}$. Determine the average distance squared from points in R to the line parallel to the z -axis and passing through the point $(0, 3, 0)$.