Combinations

Consider all of the different *groups or sets* of the letters a, b, and c by taking only 2 letters at a time and not taking order into account. They are :

$$\{ a, b \}, \{ a, c \}, \{ b, c \}$$

Note : The sets { a, b } and { b, a } are considered equal. These non-ordered groups are called combinations of the letters a, b, and c taken 2 at a time.

PROBLEMS :

1.) List all combinations of the digits 2, 4, 6, and 8 taken 2 at a time.

2.) List all combinations of the people Bill, Bonnie, Bart, Beverly, and Bob taken

a.) 2 at atime.

b.) 3 at a time.

c.) 4 at a time.

COMBINATION RULE : It can easily be shown (using permutations) that the number of combinations of n distinct objects taken k at a time is

$$C(n,k) = \frac{n!}{(n-k)! \cdot k!} .$$

PROBLEMS :

3.) Simplify the following combinations.

a.) $C(6,3)$

b.) $C(7,4)$

c.) $C(5,0)$

d.) $C(5,5)$

e.) $C(0,0)$

PROBLEMS : Use combinations to solve the following problems.

4.) A five-person entertainment committee is to be selected from 12 people. How many different committees are possible ?

5.) A committee of 10 people is to be chosen from a group consisting of 12 women and 8 men.

a.) How many different committees are possible ?

b.) How many different committees are possible if the committee must be composed of

i.) 6 women and 4 men ?

ii.) 3 women and 7 men ?

iii.) 5 women and 5 men ?

iv.) at least 7 women ?

v.) at most 5 men ?

vi.) all women ?

vii.) no women ?

6.) A 12-player basketball team is composed of 3 post players, 4 wings, and 5 guards.

a.) How many different 5-player teams are possible ?

b.) How many different 5-player teams are possible if the team has

i.) 3 wings and 2 guards ?

ii.) 2 posts and 3 guards ?

iii.) 1 post, 2 wings, and 2 guards ?

iv.) at most 2 posts ?

v.) at least 1 guard ?

vi.) no wings ?

vii.) no guards ?

7.) You have 10 friends that have offered to form two study groups with you and you get to pick both of the study groups. You get to pick 3 friends for the math study group and 5 friends for the English study group. How many distinct two-group outcomes are possible if

a.) no one can be in both study groups (except you) ?

b.) your friends can be in both study groups ?

8.) A standard deck of playing cards has 52 cards. The face values are Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, and King of the four different suits– hearts $\heartsuit$, clubs $\clubsuit$, diamonds $\diamondsuit$, and spades $\spadesuit$. Assume that you are dealt a 3-card "hand." Solutions to the following problems may require using the Fundamental Principle of Counting, permutations, or combinations.

a.) How many different hands are possible ?

b.) How many different hands, where all three cards are hearts, are possible ?

c.) How many different hands, where all three cards are clubs, are possible ?

d.) How many different hands, where all three cards are of the same suit, are possible ?

e.) How many different hands with 3 seven's are possible ?

f.) How many different hands with 3 Queen's are possible ?

g.) How many different hands with 3 of a kind of any face value are possible ?

h.) How many different hands with one pair of two's are possible ?

i.) How many different hands with one pair of Jack's are possible ?

j.) How many different hands with one pair of any face value are possible ?

k.) How many different hands with 1, 2, and 3 (of any suit) are possible ?

l.) How many different hands with Jack, Queen, and King (of any suit) are possible ?

m.) How many different hands with three consecutive cards (of any suit) are possible ?

n.) How many different hands have no cards with the same face value?

9.) In the game of Poker you are dealt 5 cards from a standard deck of 52 cards. Here is the following ranking of hands from lowest to highest–

POKER HANDS

high card

2 of a kind (1 pair of the same face value, 3 other face values)

2 pairs (2 each of two different face values, 1 other face value)

3 of a kind (3 cards of the same face value, 2 other face values)

straight (5 consecutive cards, but not all of same suit)

flush (5 cards of the same suit, but not 5 consecutive cards)

full house (2 of a kind and 3 of a kind)

4 of a kind (4 of same face value, 1 other face value)

straight flush (5 consecutive cards of the same suit)

royal flush (10, Jack, Queen, King, Ace of the same suit)

Assume that the cards are shuffled and you are dealt a 5-card hand. Solutions to the following problems may require using the Fundamental Principle of Counting, permutations, combinations, creativity, or common sense.

a.) How many different hands are possible ?

b.) How many different heart flushes are possible ?

c.) How many different flushes of any suit are possible ?

d.) How many different royal flushes of any suit are possible ?

e.) How many different straights are possible ?

f.) How many different straight flushes are possible ?

g.) How many different 4 of a kind hands are possible ?

h.) How many different full houses are possible ?

i.) How many different 3 of a kind hands are possible ?

j.) How many different 2 of a kind hands are possible ?

k.) How many different 2 pair hands are possible ?

l.) How many different high card hands are possible ?

10.) Show that $P(n,k) = C(n,k) \cdot P(k,k)$.