Freshmen Seminar
Kouba
Fundamental Counting and Combinatorics

(Some content and problems are taken from Introductory Combinatorics (3rd Edition) by R. A. Brualdi, A Transition to Advanced Mathematics (5th Edition) by D. Smith, M. Eggen, & R. St. Andre, The Nature of Modern Mathematics (2nd Edition) by K. J. Smith, and Mathematics for Elementary School Teachers (3rd Edition) by Billstein, Libeskind, and Lott.)

Combinatorics can be defined as the systematic combining, organizing, or counting of objects or processes. It is concerned with arrangements of these objects or processes into patterns satisfying special rules. Often a "best" or "optimal" arrangement is the desired outcome. It has historical roots in mathematical puzzles and games, and is an integral component of mathematics and computer science with applications in nearly any field or discipline where counting is required.

Fundamental Principle of Counting

Fundamental Principle of Counting– Assume that task A can be completed in m different ways and task B can be completed in n different ways. Then both tasks can be completed in mn different ways. (This principle is easily extended to any finite number of tasks.)

PROBLEMS :

1.) You have to pick a pair of shoes and a pair of socks to wear. You have black shoes, whites shoes and blue shoes. You have gray socks, black socks, brown socks, red socks, and white socks. How many different shoes/socks color combinations are there ?

2.) You are riding your bicycle from Davis to Woodland and then from Woodland to Winters. There are 5 different routes from Davis to Woodland.

There are 7 different routes from Woodland to Winters. How many different one-way routes are there from Davis to Winters by way of Woodland ? How many different round-trip routes are there from Davis to Winters by way of Woodland, and then returning from Winters to Davis by way of Woodland ?

3.) For fall semester you are to enroll in 5 classes– an art class, a math class, an English class, a social studies class, and a science class. There are 3 art teachers, 7 math teachers, 8 English teachers, 4 social studies teachers, and 6 science teachers from which to pick. How many different teacher assignments are possible for your 5-class schedule ?

4.) How many different outcomes are possible if you flip a coin

a.) once ?

b.) twice ?

c.) 3 times ?

d.) 5 times ?

e.) n times, where n is a natural number ?

5.) Flip a coin, pick a card, flip a coin again, roll one die, and flip a coin again. How many different outcomes are possible ?

6.) a.) How many different 1-digit numbers are there ?

b.) How many different 2-digit numbers are there ?

c.) How many different 3-digit numbers are there ?

d.) How many different 10-digit numbers are there ?

e.) How many different odd 4-digit numbers are there ?

f.) How many different even 7-digit numbers are there ?

g.) How many different 3-digit numbers are there which

i.) end in 7 ?

ii.) end in 2, 3, or 4 ?

iii.) begin with 5 and end with 0 or 1 ?

iv.) begin with an odd number and end with an even number ?

7.) In each slot of the following four-by-five table (matrix) place a number from the set $\{3, 4, 5, 6\}$. You may use a number more than once. How many distinct tables (matrices) are possible ?

8.) An ordered pair (x, y) is to be constructed by choosing x from the set $A = \{a, b, c, d\}$ and choosing y from the set $B = \{-2, -1, 0, 1, 2\}$. How many different ordered pairs are possible ?

9.) An ordered triple (x, y, z) is to be constructed by choosing $x, y,$ and z from the set B in problem 8. How many different ordered triples are possible?

10.) Some California license plates are of the form 2XPK914, that is, a digit $(0, 1, 2, \cdots, 9)$ followed by 3 letters, followed by 3 numbers. How many different license plates of this design are possible ?

11.) One die has six faces with each of the numbers 1, 2, 3, 4, 5, and 6 on a face. Roll one die twice. How many different outcomes are possible ? Roll the die twice and sum the two numbers that result. How many ways can the sum be

a.) one ?

b.) two ?

c.) three ?

d.) four ?

e.) five ?

f.) six ?

g.) seven ?

h.) eight ?

i.) nine ?

j.) ten ?

k.) eleven ?

l.) twelve ?

m.) thirteen ?

12.) Juanita, Jim, Jennifer, Jammal, Jesus, and Julia are running for seats on the student council at the local high school. The available positions are president, vice-president, secretary, and treasurer. If no person can hold more than one position, how many different election outcomes are possible ?

13.) You are to make secret code "words" from the following set of symbols : $\{\cap, \cup, \triangle, \#, \oplus\}$. For example, $\triangle\#\#$ (ice cream) and $\cap\cup\oplus\triangle$ (trampoline) are words.

a.) How many different 1-character words can be created ?

b.) How many different 2-character words can be created if

i.) all of the characters in a word must be different ?

ii.) the characters in a word may be repeated ?

c.) How many different 3-character words can be created if

i.) all of the characters in a word must be different ?

ii.) the characters in a word may be repeated ?

d.) How many different 4-character words can be created if

i.) all of the characters in a word must be different ?

ii.) the characters in a word may be repeated ?

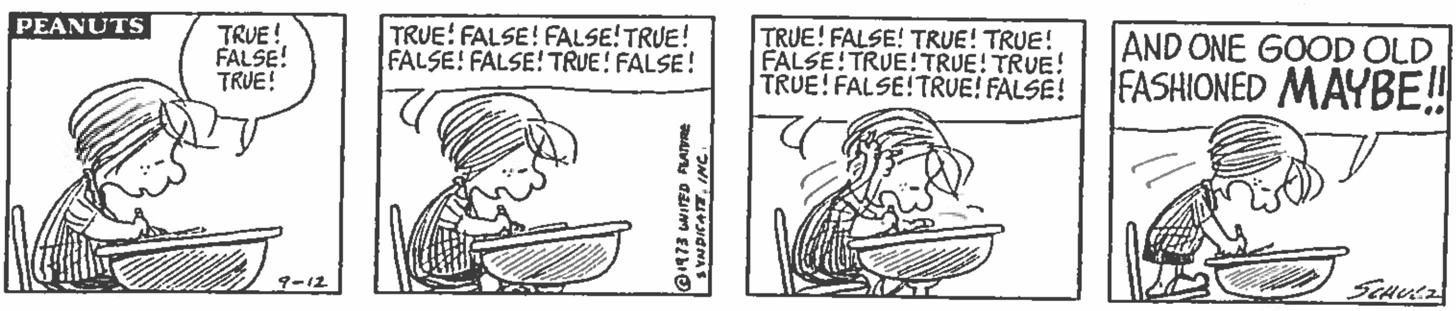
e.) How many different 5-character words can be created if

i.) all of the characters in a word must be different ?

ii.) the characters in a word may be repeated ?

f.) How many different words do there exist if we count all words of any length up to and including 30-character words and if we allow words with repeated characters ?

14.) A true/false (T or F, no maybe's) exam has 5 questions. If you guess the answer on every question, how many different answer sheets are possible ? If the exam has 25 questions, how many different answer sheets are possible ? If the exam has n questions, where n is a natural number, how many different answer sheets are possible ?



15.) Let A be a nonempty set. Then set B is a subset of A if every element of B is also in A. For example, if $A = \{1, 2, 3\}$, then $B = \{2, 3\}$ is a subset of A. We write $B \subseteq A$. List all of the subsets for each of the following sets A . (Note that the empty set is a meaningful set. It is denoted by $\{ \}$ or ϕ. Assume that the empty set is a subset of every set.)

a.) $A = \{1\}$

b.) $A = \{1, 2\}$

c.) $A = \{1, 2, 3\}$

d.) $A = \{1, 2, 3, 4\}$

e.) How many subsets does $A = \{1, 2, 3, 4, 5\}$ have ? You need NOT list the subsets. HINT: Use the Fundamental Principle of Counting.

f.) How many subsets does $A = \{1, 2, 3, ..., 10\}$ have ? You need NOT list the subsets. HINT: Use the Fundamental Principle of Counting.

g.) How many subsets does $A = \{1, 2, 3, ..., n\}$ have, where n is a natural number ? You need NOT list the subsets. HINT: Use the Fundamental Principle of Counting.

16.) Jose and Ahmad are two boys who will sit in a row of 5 chairs with the girls Lisa, Fatima, and Yilda. How many ways can this be done if

a.) the boys and girls can sit anywhere ?
b.) the boys must sit together ?
c.) the girls must sit together ?
d.) the boys must sit together and the girls must sit together ?
e.) Jose and Yilda must sit together ?