Permutations of Letters in a Word

Consider all of the distinct *permutations* of the letters in the word *can* . They are :

$$acn, anc, can, nac, cna, nca$$

There are $P(3,3) = \frac{3!}{0!} = \frac{3 \cdot 2 \cdot 1}{1} = 6$ of them. Consider all of the distinct *permutations* of the letters in the word *moon* . We showed in problem 9 in the section on permutations that they are :

$$moon, oomn, mnoo, noom, oonm, nmoo,$$

$$omno, onmo, omon, onom, mono, nomo$$

There are NOT $P(4,4) = \frac{4!}{0!} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{1} = 24$ of them. There are only 12 of them. Where did the other 12 go ? They disappeared because the four letters in this word are not all different. The letter *o* is repeated. The following problems may help us recognize patterns and formulate a way to determine the permutations of the letters in a word, where some of the letters are repeated.

PROBLEMS :

1.) Make a careful and systematic list of all distinct permutations of the letters in the following words.

a.) *me*

b.) *mit*

c.) *boo*

d.) *dude*

e.) *motto*

f.) *bababa*

2.) Without making a list of all of them, use combinations (or the Fundamental Principle of Counting) to count all possible distinct permutations of the letters in the following words.

a.) *bib*

b.) *zoom*

c.) *lluuull*

d.) *googol*

e.) *motto*

f.) *hahaha*

g.) *bowwowwow*

LETTER PERMUTATION RULE : Assume that you have an n-letter word composed of letters, some of which are repeated. Assume that there are m letters appearing more than once. These letters are

$$a_1, a_2, a_3, \ldots\ a_m,$$

and each letter appears

$$k_1, k_2, k_3, \ldots\ k_m \text{ times, respectively.}$$

Then the number of *distinct permutations of these n letters* is

$$\frac{n!}{k_1! \cdot k_2! \cdot k_3! \cdots k_m!}$$

Example : The number of distinct permutations of the letters in the word *lluuull* is $\dfrac{7!}{4! \cdot 3!} = \dfrac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(4 \cdot 3 \cdot 2 \cdot 1)(3 \cdot 2 \cdot 1)} = 35$.

PROBLEMS :

3.) Determine the number of distinct permutations of the letters in the following words.

a.) *radar*

b.) *cannonball*

c.) *boomboom*

d.) *Mississippi*

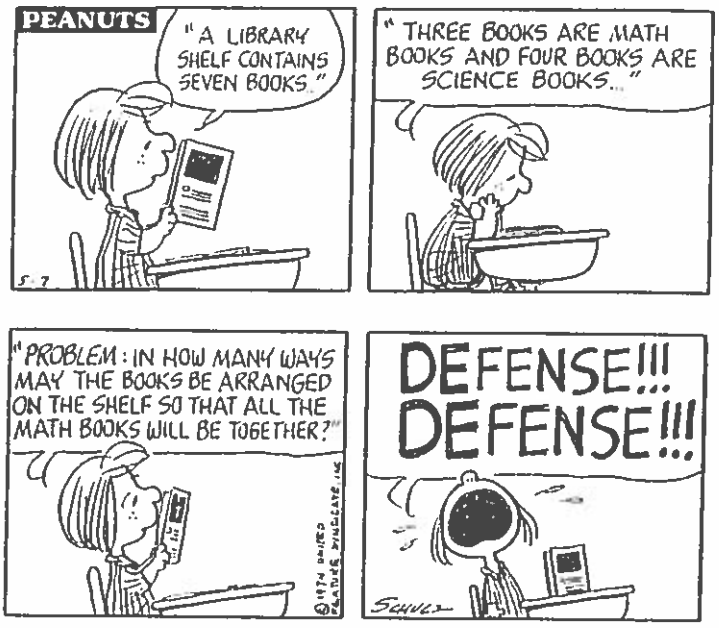
e.) *onomatopoeia*

f.) *wwxxxyzzzzxwyz*

4.) Can you calm Peppermint Patti down and help her solve the problem given below by

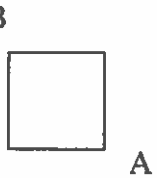
a.) assuming that the math books are exactly the same and the science books are exactly the same ?

b.) assuming that the math books are all distinct and the science books are all distinct ?



5.) Count one-by-one all of the distinct "shortest" paths from point A to point B in the following grids. In other words, you can only travel "up" and "left" along the grid lines.

a.)



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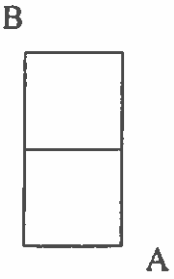

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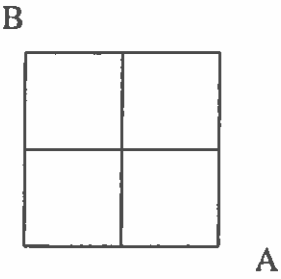
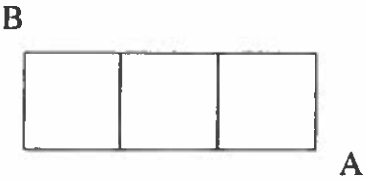
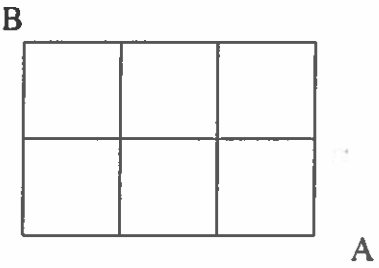


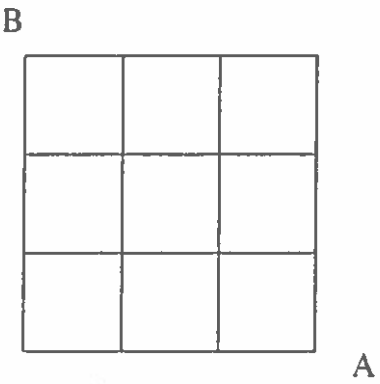
d.)



e.)

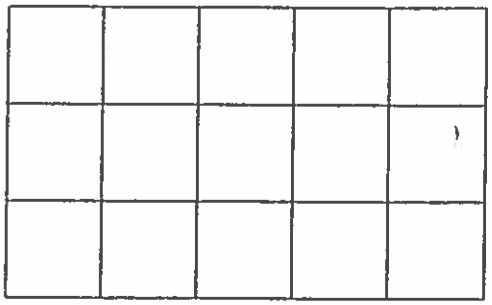


f.)



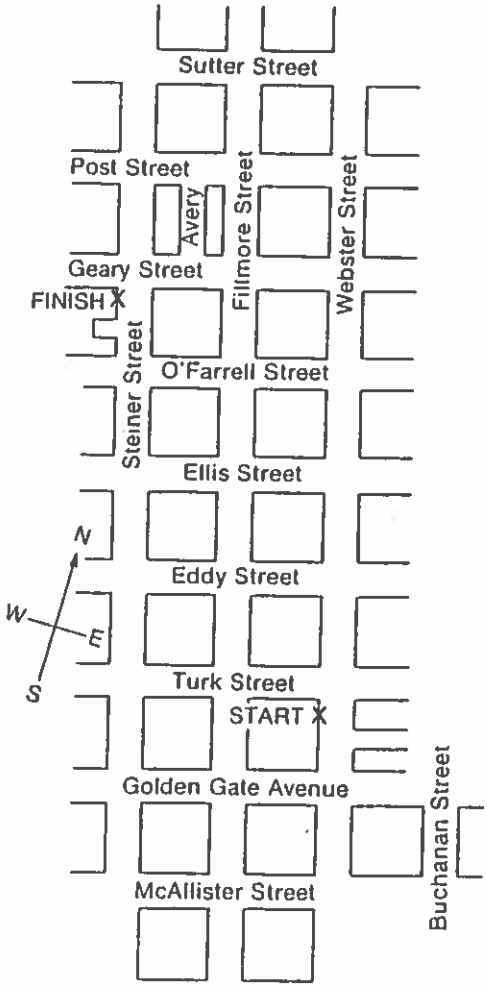
6.) SHORTCUT FOR COUNTING PATHS : By moving only up (U) or left (L) we wish to count all of the possible paths from point A to point B in the given three-by-five grid. Since there are five left moves (5 L's) and three up moves (3 U's) required, we can think of this task as determining the number of permutations of the letters in the "word" LLLLLUUU. The answer is $\frac{8!}{5! \cdot 3!} = 56$!

B



A

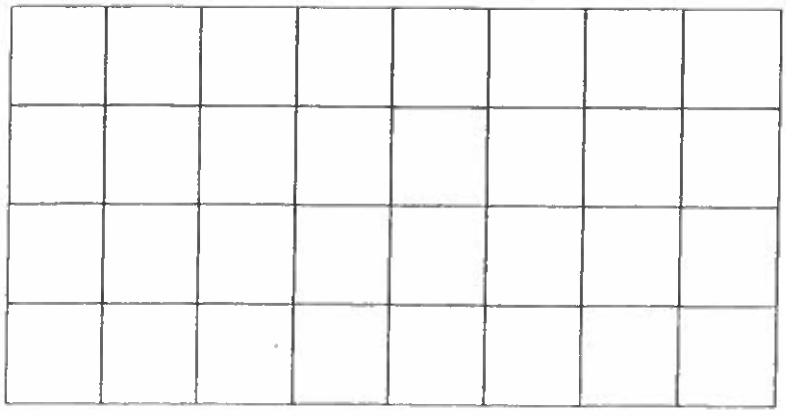
7.) In how many ways can a person in San Francisco walk from the corner of Webster and Turk Streets to the corner of Geary and Steiner Streets by traveling exactly six blocks ?



8.) How many ways can you get from point A to point B in the following grids by traveling only up or left along grid lines ?

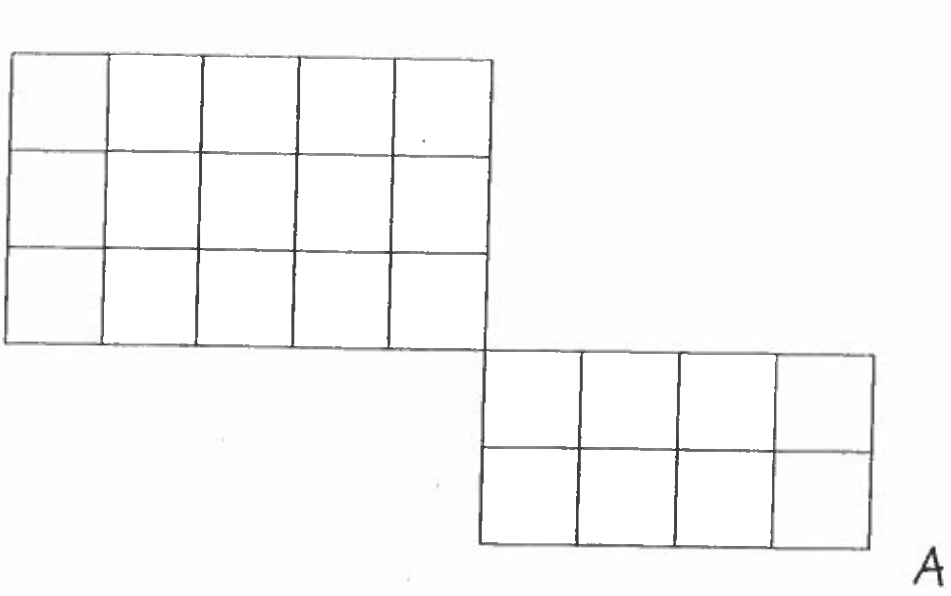
a.)

B



A

b.)



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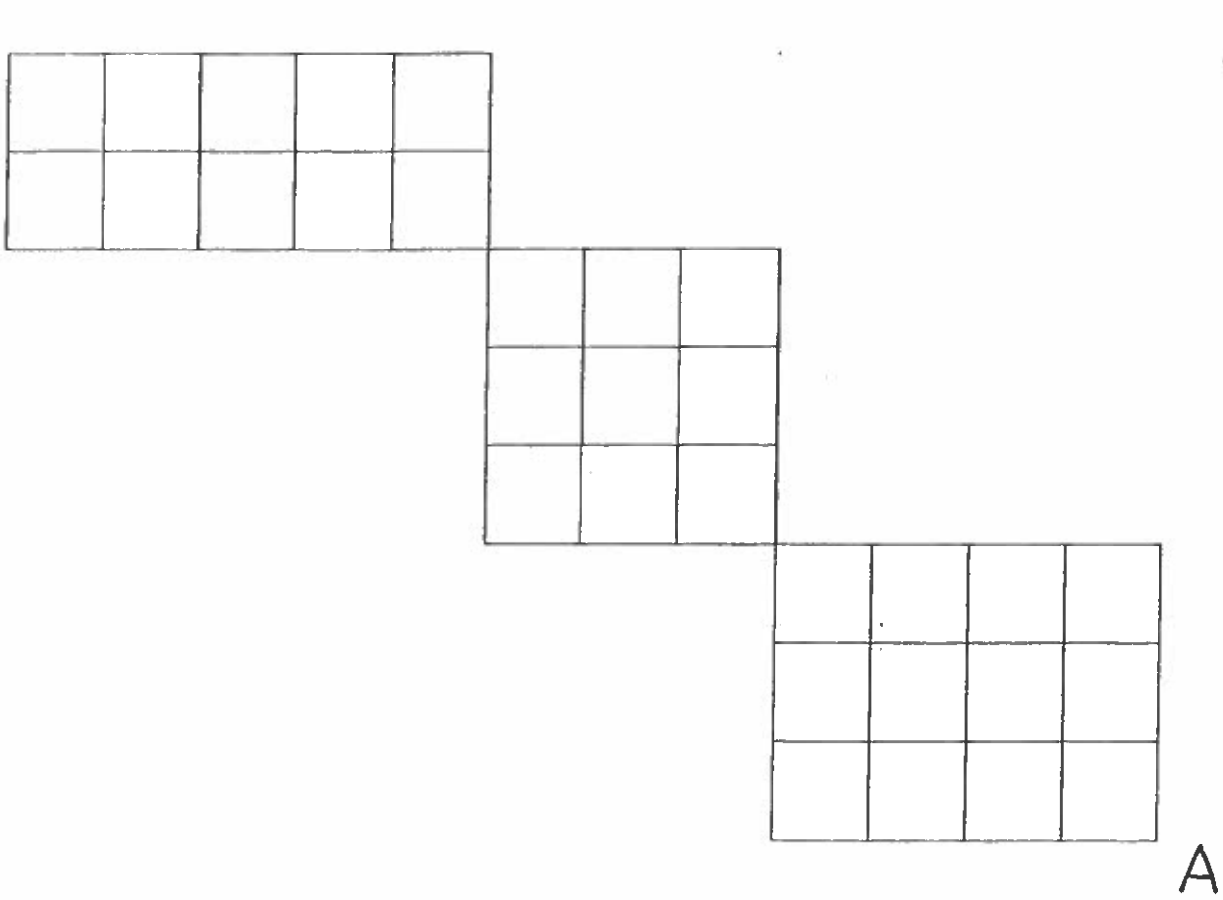

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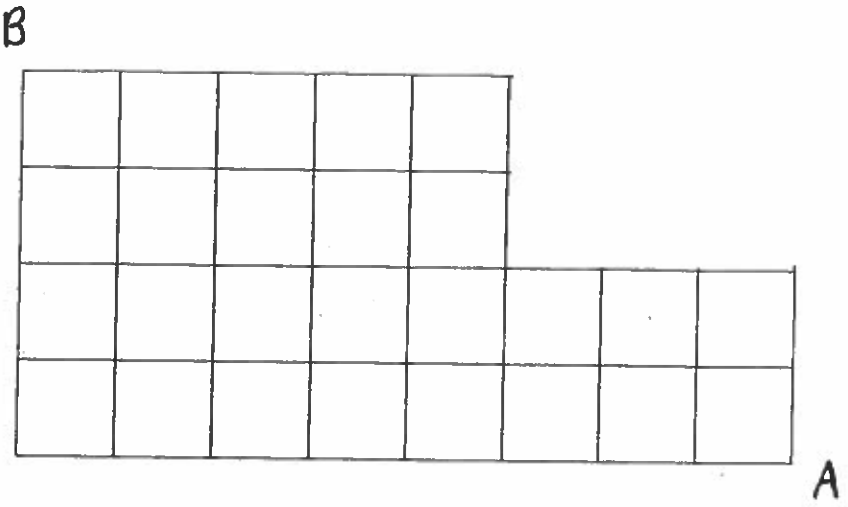
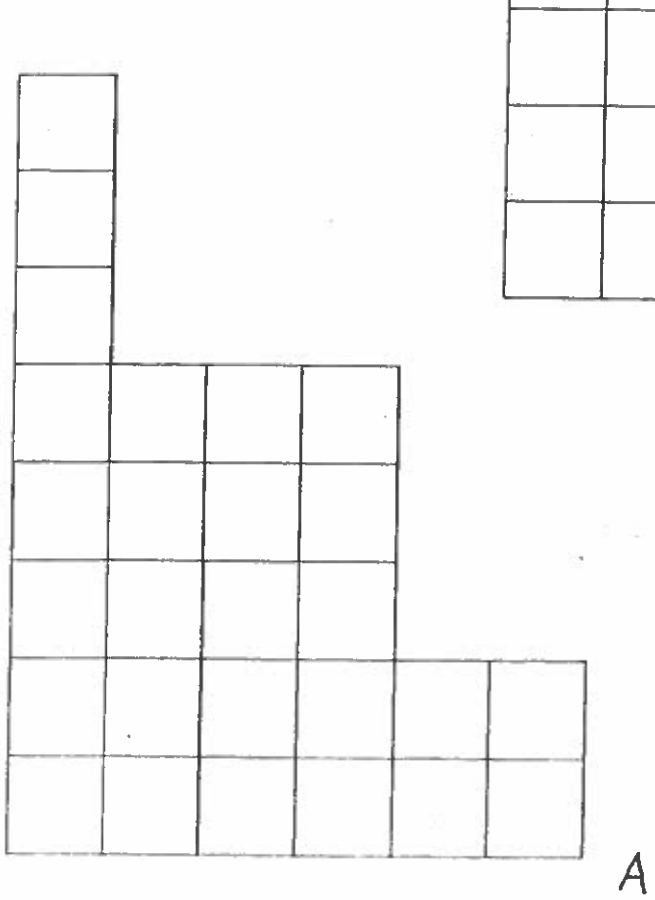


e.)



f.)