Operations on Sets
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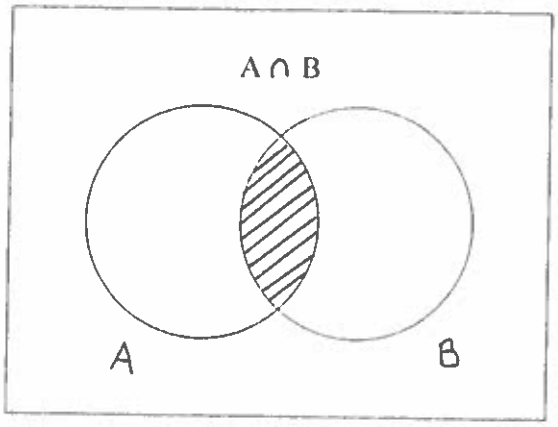
The following formal operations on two sets will allow us to create new sets. Together, these sets and operations provide another tool with which to solve mathematics problems.

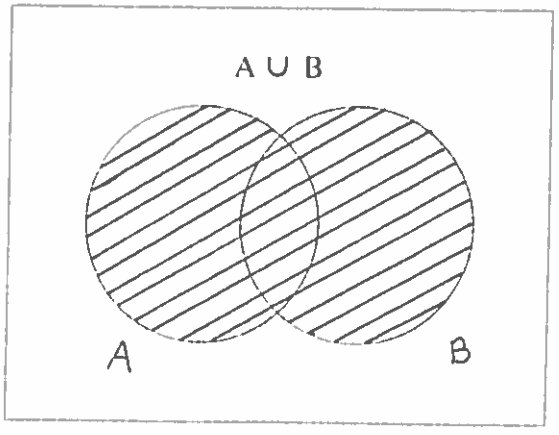
Definition : The *intersection* of two sets A and B, written $A \cap B$, is the set of all elements common to both A and B, i.e.,

$$A \cap B = \{x : x \in A \text{ and } x \in B\} .$$



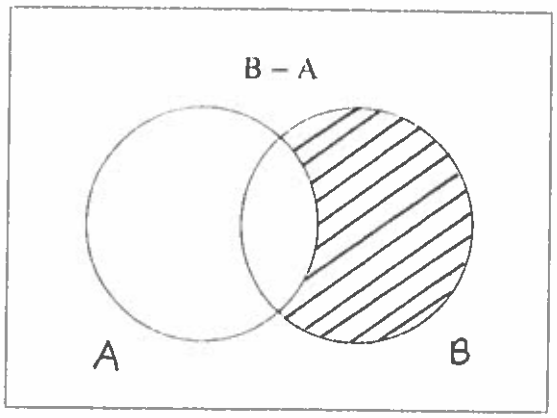
The *union* of two sets A and B, written $A \cup B$, is the set of all elements in A or in B or in both A and B, i.e.,

$$A \cup B = \{x : x \in A \text{ or } x \in B\} .$$



The *complement of set A relative to set B* , written $B - A$, is the set of all elements in B that are not in A, i.e.,

$$B - A = \{x : x \in B \text{ and } x \notin A\} .$$



Example : Let set $A = \{1, 3, 4, 9\}$ and set $B = \{3, 4, 7\}$. Then $A \cap B = \{3, 4\}$, $A \cup B = \{1, 3, 4, 7, 9\}$, $B - A = \{7\}$, and $A - B = \{1, 9\}$.

PROBLEMS :

1.) Determine $A \cap B$, $A \cup B$, $B - A$, and $A - B$ for each pair of sets A and B.

a.) $A = \{a, b, c, d\}$ and $B = \{c, d, e\}$

b.) $A = \{a, b, c, d\}$ and $B = \{b, c\}$

c.) $A = \{a, b\}$ and $B = \{c, d\}$

d.) $A = \{a, b, c, d\}$ and $B = \{\ \}$

e.) $A = \{2, 4, 6, 8\}$ and $B = \{2, 4, 6, 8\}$

f.) $A = \{1, 2, 3, 4, 5, \cdots\}$ and $B = \{1, 2, 3, 4, 5, 6, 7, 8\}$

g.) $A = \{1, 2, 3, 4, 5, \cdots\}$ and $B = \{2, 4, 6, 8, 10, \cdots\}$

h.) $A = \{1, 1/2, 1/3, 1/4, 1/5, \cdots\}$ and $B = \{0, 1/100, 1\}$

i.) $A = \{\triangle, \heartsuit, \surd, \infty\}$ and $B = \{\triangle, \heartsuit, \infty\}$

2.) In a sorority with 30 members, 18 take mathematics, 5 take both mathematics and biology, and 8 take neither mathematics nor biology. How many take biology but not mathematics ?

3.) Of the 11th-graders at Paxson High School, there were 7 who played basketball, 9 who played volleyball, 10 who played soccer, 1 who played basketball and volleyball only, 1 who played basketball and soccer only, 2 who played volleyball and soccer only, and 2 who played volleyball, basketball, and soccer. How many played one or more of the three sports ?

4.) At the end of a tour of the Grand Canyon, several guides were talking about the people on the latest British-American tour. The guides could not remember the total number in the group. However, together they compiled the following statistics about the group. It contained 26 British females, 17 American women, 17 American males, 29 girls, 44 British citizens, 29 women, and 24 British adults. How many British boys were in the group ? What was the total number of people in the group ?

5.) Howie, O. J., and Frank each predicted winners for one Sunday's NFL professional football games. The only team not picked to be a winner by any of the sportscasters that Sunday was the Giants. The choices for winners for each person were as follows :

Howie : Cowboys, Steelers, Vikings, Bills
O. J. : Steelers, Packers, Cowboys, Redskins
Frank : Redskins, Vikings, Jets, Cowboys

If the only teams playing that Sunday were those listed, which teams played each other that day ?

6.) A paper carrier in Rattlesnake, Texas, delivers 31 copies of the Town Gazette and 37 copies of the Daily Flyer each day to at most 60 houses. If no house receives 2 copies of the same paper, answer the following questions.

a.) How many ways can the Gazette be delivered to 50 houses ?
b.) How many ways can the Flyer be delivered to 45 houses ?
c.) How many ways can all of the Gazettes and Flyers be delivered to 60 houses ?
d.) What is the greatest number of houses to which 2 papers can be delivered ?

e.) What is the least number of houses to which 2 papers can be delivered?

Assume that a new newspaper, the Democratic Republican, begins delivering 45 copies each day in addition to the Flyer and the Gazette.

f.) What is the greatest number of houses to which 3 papers can be delivered ?

g.) Assume that 18 houses get the Gazette and the Flyer, 28 houses get the Flyer and the Republican, 19 houses get the Gazette and the Republican, and 5 houses get the Gazette only.

i.) How many houses get all three newspapers ?
ii.) How many houses get the Flyer only ?
iii.) How many houses get no newspapers ?