Sets
D. A. Kouba

Formulas and equations are an important part of algebra, trigonometry, analytic geometry, calculus, and advanced mathematics classes. Certain equations, called functions, become the focus of work involving graphing, differentiation, or integration. Functions can often be characterized by their graphs in the Cartesian plane. In this context, functions can be considered as sets of ordered pairs $(x, y)$ in two-dimensional space. We begin this discussion of sets with some basic definitions.

## Sets

Definition : A *set* is a collection of objects or elements. If $x$ is an element in set $A$, we write $x \in A$. If $x$ is not an element in set $A$ we write $x \notin A$.

Example : Let set $A = \{5, 6\}$. Then $5 \in A$ and $6 \in A$.

Example : Here is a list of different types of sets.

1.) $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

2.) $\{e, f, g\}$

3.) $\{red, yellow, blue, green\}$

4.) $\{(1, 1), (2, 3), (4, 5), (-3, 2), (0, 0)\}$

5.) $\{2, 4, 6, 8, 10, \cdots\}$

6.) $\{ \quad \}$ ; this set is called the *empty set* . It is meaningful, just empty. The empty set can also be represented by $\phi$ .

7.) $\{\phi, \{3\}, \{4, 5\}\}$

8.) $\{x : x^2 = 9 \text{ and } x \text{ is an integer } \} = \{-3, 3\}$

Definition : Let $A$ be a set. Set $B$ is a *subset* of set $A$ if every element in $B$ is also in $A$. We write $B \subseteq A$. If $B$ is not a subset of $A$ we write $B \nsubseteq A$.

Note : We will assume that the empty set $\phi$ is a subset of *every* set $A$ and every set $A$ is a subset of itself, i.e.,

1.) $\phi \subseteq A$ for all sets $A$

and

2.) $A \subseteq A$ for all sets $A$ .

Example : The following examples illustrate the proper use of set notation.

1.) $\{1, 2\} \subseteq \{1, 2, 3, 4\}$

2.) $\{red\} \subseteq \{red, green\}$

3.) $\{yellow, blue, green, white\} \nsubseteq \{red, yellow, blue, green\}$

4.) $\{1, 2, 3, 4, 5, \cdots\} \subseteq \{\cdots, -3, -2, -1, 0, 1, 2, 3, \cdots\}$

5.) $\{q, r, s, t\} \subseteq \{q, r, s, t\}$

6.) $\{\{1\}\} \subseteq \{\{1\}, \{2\}\}$

7.) $\{a, b, c\} \nsubseteq \{b, c, d, e\}$

8.) $\{1\} \in \{\{1\}, \{2\}\}$

9.) $1 \notin \{\{1\}, \{2\}\}$

10.) $\{1\} \nsubseteq \{\{1\}, \{2\}\}$

11.) $\phi \subseteq \{\phi, \{2\}\}$

12.) $\phi \in \{\phi, \{2\}\}$

PROBLEMS : Find all of the subsets for each set $A$ .

1.) $A = \phi$

2.) $A = \{1\}$

3.) $A = \{x, y\}$

4.) $A = \{B, C, D\}$

5.) $A = \{\triangle, \heartsuit, \surd, \infty\}$

Fact : (The following statement can be easily proven using the Fundamental Principle of Counting.) If set $A$ has $n$ elements, then set $A$ has $2^n$ subsets.

PROBLEMS : Determine the number of the subsets for each set $A$ .

6.) $A = \{5, 6, 7, 8, 9, 10, 11, 12, 13\}$

7.) $A = \{26, 27, 28, 29, 30, \cdots, 71, 72\}$

8.) $A = \{5, 9, 13, 17, \cdots, 129, 133\}$

9.) $A = \{\{1\}, \{2\}, \{3\}\}$

10.) $A = \{(0, 0), (1, 1), (2, 2), (3, 3), (4, 4)\}$

Definition : Let $A$ be a set. The *power set* of $A$ is the set of all subsets of $A$ and is denoted $P(A)$, i.e.,

$$P(A) = \{B : B \subseteq A\} .$$

Example : If $A = \{u, v\}$, then $P(A) = \{\phi, \{u\}, \{v\}, \{u, v\}\}$ .

Note : If set $A$ has $n$ elements, then a previous fact guarantees that the number of elements in the power set $P(A)$ is $2^n$.

PROBLEMS :

11.) Determine the power set $P(A)$ for each set $A$ .

a.) $A = \phi$

b.) $A = \{1\}$

c.) $A = \{3, \{3\}\}$

d.) $A = P(S)$, where $S = \{a, b\}$

12.) If $A = \{v, w, x, y, z\}$ and $B$ is the set of all subsets of $A$ which contain an even number of elements, then how many elements are in $P(B)$?

13.) If $A = \{a, b, c, d, e, f, g, h, i, j\}$ and $B$ is the set of all subsets of $A$ which contain an odd number of elements, then how many elements are in $P(B)$?

14.) Determine a set $A$ so that $A \subseteq P(A)$, or explain that this is impossible.

15.) Determine a set $A$ so that $A \in A$, or explain that this is impossible.