Def: Function \( Y = f(x) \) is continuous at \( x = a \) if

1. \( f(a) \) exists (finite number)
2. \( \lim_{{x \to a}} f(x) \) exists (finite number)
3. \( \lim_{{x \to a}} f(x) = f(a) \)

Fact: Sums, differences, products, quotients (denominator \( \neq 0 \)), and compositions of continuous functions are continuous.

Fact: Every polynomial is continuous for all \( x \)-values.

Ex: Let \( f(x) = 7x^5 - x^4 + 2x^3 - x + 20 \); \( f \) is continuous for all values of \( x \) since it is a polynomial.

Ex: Let \( f(x) = \frac{x^2 - 5x + 6}{2x^2 + x - 3} \); since \( Y = x^2 - 5x + 6 \) (parabola) and \( Y = 2x^2 + x - 3 \) (parabola) are continuous for all \( x \)-values, it follows that \( f(x) = \frac{x^2 - 5x + 6}{2x^2 + x - 3} \) (quotient) is continuous for all \( x \)-values except where \( Y = 2x^2 + x - 3 = (2x+3)(x-1) = 0 \), that is, except at \( x = 1 \) and \( x = -\frac{3}{2} \).

Ex: Let \( f(x) = (3 + \sin x)^{50} \); since \( g(x) = 3 + \sin x \) (well-known continuous function) and \( h(x) = x^{50} \) (polynomial) are continuous for all values of \( x \), it follows that their composition \( f(x) = h(g(x)) = (3 + \sin x)^{50} \) is continuous for all values of \( x \).