Math 16A (Fall 2020)
Kouba
Exam 1

Printing and signing your name below is a verification that no other person assisted you in the completion of this Exam.

PRINT your name ______________________ SIGN your name KEY

Show clear, organized supporting work for your answers. Correct answers without supporting work may not receive full credit. Use of unapproved shortcuts may not receive full credit. There are 13 pages. You must submit exactly 13 pages to Gradescope.

1.) Determine the following limits.

a.) (10 pts.) $\displaystyle\lim_{x \to 2} \frac{x^3 - 8}{x^2 - 4}$

$$\overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x \to 2} \frac{\cancel{(x-2)}(x^2+2x+4)}{\cancel{(x-2)}(x+2)}$$

$$= \frac{4+4+4}{4} = 3$$

b.) (10 pts.) $\lim\limits_{x\to-1} \dfrac{x+1}{\sqrt{x+5}-2} \cdot \dfrac{\sqrt{x+5}+2}{\sqrt{x+5}+2}$

$$\overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to-1} \frac{(x+1)(\sqrt{x+5}+2)}{(x+5)-4}$$

$$= \lim_{x\to-1} \frac{\cancel{(x+1)}(\sqrt{x+5}+2)}{\cancel{x+1}} = \sqrt{4}+2 = 4$$

c.) (10 pts.) $\lim_{x \to 0} \dfrac{\frac{1}{x-2} + \frac{1}{x+2}}{x}$

$$\text{"}\tfrac{0}{0}\text{"} \quad \lim_{x \to 0} \frac{(x+\cancel{2}) + (x-\cancel{2})}{(x-2)(x+2)} \cdot \frac{1}{x}$$

$$= \lim_{x \to 0} \frac{2\cancel{x}}{\cancel{x}(x-2)(x+2)} = \frac{2}{(-2)(2)} = -\frac{1}{2}$$

d.) (10 pts.) $\lim_{x\to-\infty} \frac{2x^2-x}{3x+5} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} \lim_{x\to-\infty} \frac{2x-1}{3+5/x}$

$$= \frac{-\infty-1}{3+0} = -\infty$$

2.) a.) (8 pts.) Determine the domain of $f(x) = \dfrac{x-1}{\sqrt{x^2-4x}}$.

Need: $x^2 - 4x = x(x-4) > 0$

$+ \quad 0 \quad - \quad 0 \quad +$

$x=0 \qquad x=4$

Domain: $x < 0, \ x > 4$

b.) (8 pts.) Determine the range of $f(x) = 2 - 3\sin x$.

$-1 \le \sin x \le +1$

$\rightarrow -3 \le -3\sin x \le 3$

$\rightarrow -1 \le \underbrace{2-3\sin x}_{y} \le 5$

$\rightarrow$ <u>Range</u>: $-1 \le Y \le 5$

3.) (9 pts.) Prove algebraically that the function $f(x) = \dfrac{x+1}{3-x}$ is one-to-one, i.e., show that if $f(x_1) = f(x_2)$, then $x_1 = x_2$.

$$f(x_1) = f(x_2) \rightarrow \frac{x_1+1}{3-x_1} = \frac{x_2+1}{3-x_2}$$

$$\rightarrow (x_1+1)(3-x_2) = (x_2+1)(3-x_1)$$

$$\rightarrow 3x_1 + \cancel{3} - \cancel{x_1x_2} - x_2$$

$$= 3x_2 + \cancel{3} - \cancel{x_1x_2} - x_1$$

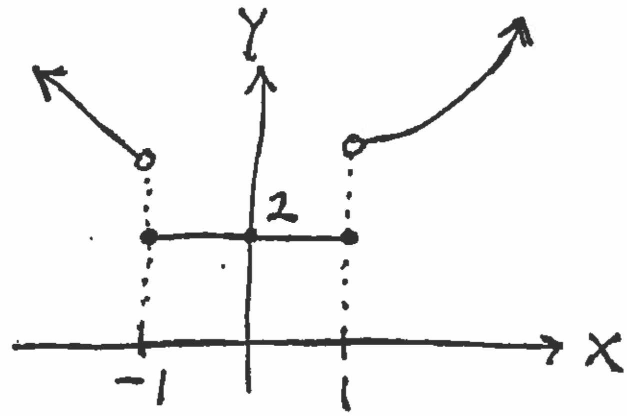
$$\rightarrow 4x_1 = 4x_2$$

$$\rightarrow x_1 = x_2 \text{, so } f \text{ is 1-1.}$$

4.) (12 pts.) Use LIMITS and a FAKE GRAPH to determine the value of constants A and B so that the following function is continuous for all values of x :

$$f(x) = \begin{cases} Ax + B\ , & \text{if } x < -1 \\ 2\ , & \text{if } -1 \le x \le 1 \\ x^2 + 3Bx + A\ , & \text{if } x > 1 \end{cases}$$

$$\left.\begin{array}{l} \lim\limits_{x \to 1^+} (x^2 + 3Bx + A) = 2 \\ \lim\limits_{x \to -1^-} (Ax + B) = 2 \end{array}\right\} \rightarrow$$



$$\left.\begin{array}{l} 1 + 3B + A = 2 \\ -A + B = 2 \end{array}\right\} \rightarrow$$

$$\left.\begin{array}{l} A = 1 - 3B \\ A = B - 2 \end{array}\right\} \rightarrow 1 - 3B = B - 2 \rightarrow 4B = 3$$

$$\rightarrow \boxed{B = \frac{3}{4}} \text{ and } \boxed{A =} B - 2 = \frac{3}{4} - \frac{8}{4} \boxed{= \frac{-5}{4}}$$

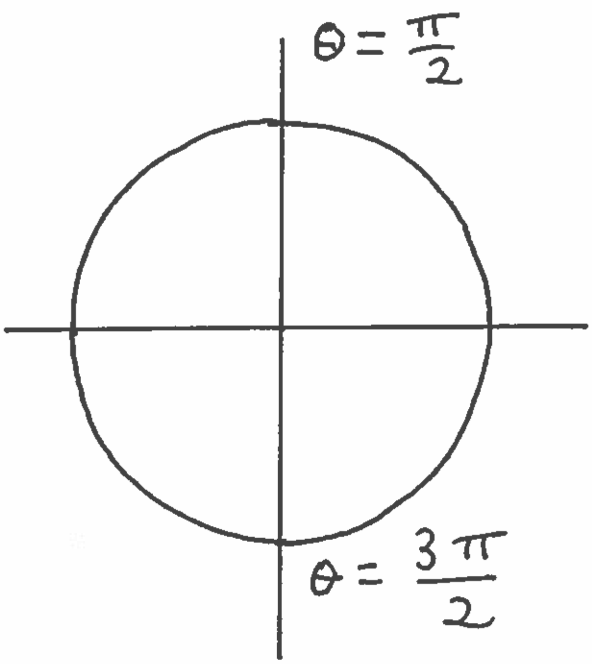
5.) (8 pts.) Solve the following trigonometry equation for θ , $0 \leq \theta \leq 2\pi$:
$\sin 2\theta - \sqrt{3}\cos\theta = 0$

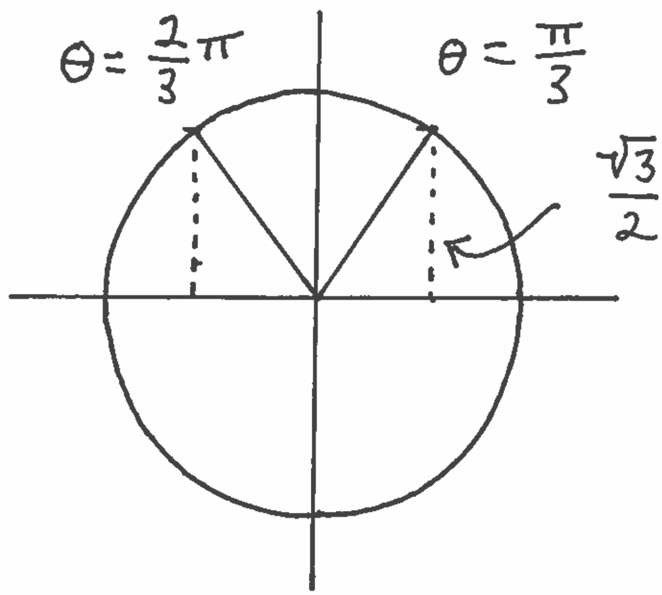
$$\sin 2\theta - \sqrt{3}\cos\theta = 0 \longrightarrow$$

$$2\sin\theta\cos\theta - \sqrt{3}\cos\theta = 0 \longrightarrow$$

$$\cos\theta\,(2\sin\theta - \sqrt{3}) = 0 \longrightarrow$$

$$\cos\theta = 0 \qquad\qquad \sin\theta = \frac{\sqrt{3}}{2}$$





$$\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{3}, \frac{2}{3}\pi$$

6.) (10 pts.) *Use limits* to find the equation(s) for all vertical asymptote(s) for $y = \dfrac{x^2 - 4x + 3}{x^2 - 9}$. YOU NEED NOT GRAPH THE FUNCTION.

$$x^2 - 9 = (x-3)(x+3) = 0$$

$$\underset{x=3}{\downarrow} \qquad \underset{x=-3}{\downarrow}$$

$$\lim_{x \to 3} \frac{x^2 - 4x + 3}{x^2 - 9} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x \to 3} \frac{\cancel{(x-3)}(x-1)}{\cancel{(x-3)}(x+3)} = \frac{2}{6} = \frac{1}{3},$$

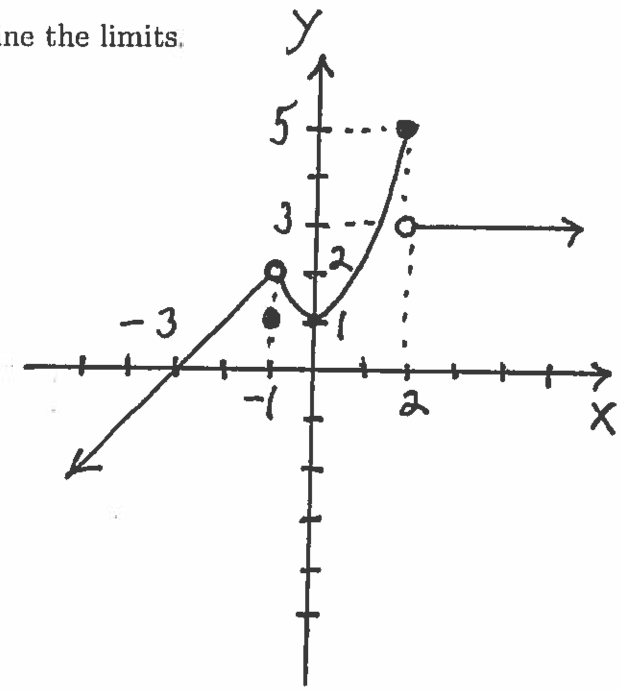
so NO V.A. ;

$$\lim_{x \to -3} \frac{x^2 - 4x + 3}{x^2 - 9} = \lim_{x \to -3} \frac{x-1}{x+3} = \frac{-4}{0} = \pm\infty$$

so V.A. is $\boxed{x = -3}$

7.) (12 pts.) Sketch the following function. Determine the limits.

$$f(x) = \begin{cases} 3\,, & \text{if } x > 2 \\ x^2+1\,, & \text{if } -1 < x \leq 2 \\ 1\,, & \text{if } x = -1 \\ x+3\,, & \text{if } x < -1 \end{cases}$$



a.) $\lim_{x \to 2^+} f(x) = 3$

b.) $\lim_{x \to 2} f(x)$ D.N.E.

c.) $\lim_{x \to -1} f(x) = 2$

d.) $\lim_{x \to -100} f(x) = \lim_{x \to -100} (x+3) = -100 + 3$

$= -97$

8.) (8 pts.) Determine the center and radius of the following circle : $x^2 + y^2 - 2x + 4y = 4$

$\rightarrow (x^2 - 2x) + (y^2 + 4y) = 4$

$\rightarrow (x^2 - 2x + 1) + (y^2 + 4y + 4) = 4 + 1 + 4$

$\rightarrow (x-1)^2 + (y+2)^2 = 9 = 3^2$

$\rightarrow$ center : $(1, -2)$ and

radius $r = 3$ ..

9.) (10 pts.) Determine the domain of the following function.

$$f(x) = \frac{\sqrt{x^2 - 2x}}{\sqrt{4x-5} - \sqrt{16-x^2}}$$

$x^2 - 2x = x(x-2) \geq 0$

$+ \quad 0 \quad - \quad 0 \quad +$

$x=0 \qquad x=2$

$4x - 5 = 4\left(x - \frac{5}{4}\right) \geq 0$

$- \quad 0 \quad +$

$x = 5/4$

$16 - x^2 = (4-x)(4+x) \geq 0$

$- \quad 0 \quad + \quad 0 \quad -$

$x=-4 \qquad x=4$

$\sqrt{4x-5} = \sqrt{16-x^2} \rightarrow 4x - 5 = 16 - x^2$

$\rightarrow x^2 + 4x - 21 = (x-3)(x+7) = 0$

<u>Domain</u>: $2 \leq x \leq 4$ and $x \neq 3$