

Section 2.8

16.) $Y = (1+x^2)^{-1} \rightarrow \frac{dY}{dt} = -(1+x^2)^{-2} \cdot 2x \cdot \frac{dx}{dt},$

$\frac{dx}{dt} = 2 \text{ cm./min.}$ so $\frac{dY}{dt} = -(1+x^2)^{-2} \cdot 2x \cdot 2$ or

$$\frac{dY}{dt} = \frac{-4x}{(1+x^2)^2}$$

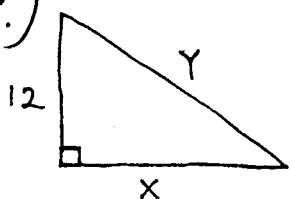
a.) If $x = -2$ then $\frac{dY}{dt} = \frac{-4(-2)}{(1+4)^2} = \frac{8}{25} \text{ cm./min.}$

b.) If $x = 2$ then $\frac{dY}{dt} = \frac{-4(2)}{(1+4)^2} = \frac{-8}{25} \text{ cm./min.}$

c.) If $x = 0$ then $\frac{dY}{dt} = \frac{-4(0)}{(1+0)^2} = 0 \text{ cm./min.}$

d.) If $x = 10$ then $\frac{dY}{dt} = \frac{-4(10)}{(1+100)^2} = \frac{-40}{10,201} \text{ cm./min.}$

18.)

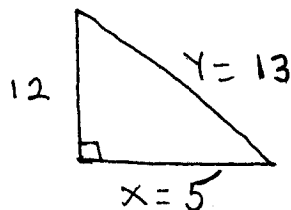


$$x^2 + 12^2 = Y^2 \text{ and } \frac{dY}{dt} = -4 \text{ ft./sec.}$$

find $\frac{dx}{dt}$:

$$2x \cdot \frac{dx}{dt} = 2Y \cdot \frac{dY}{dt} \rightarrow$$

a.)



$$(5) \frac{dx}{dt} = (13)(-4) \rightarrow$$

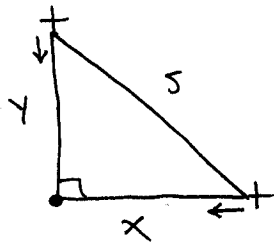
$$\frac{dx}{dt} = \frac{(13)(-4)}{5} = -\frac{52}{5} \text{ ft./sec.}$$

b.)

$$\frac{dx}{dt} = \frac{Y}{x} \left(\frac{dY}{dt} \right) = \frac{-4Y}{x} \text{ and as } x \rightarrow 0, Y \rightarrow 12$$

so $\frac{dx}{dt}$ approaches $\frac{-48}{0} = -\infty \text{ ft./sec.}$

19.)



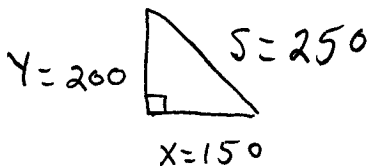
$$x^2 + y^2 = s^2 \rightarrow$$

$$\cancel{2x} \frac{dx}{dt} + \cancel{2y} \frac{dy}{dt} = \cancel{2s} \frac{ds}{dt} ;$$

a.) Find $\frac{ds}{dt}$ when $x = 150$ mi., $\frac{dx}{dt} = -450$ mph.,

$y = 200$ mi., and $\frac{dy}{dt} = -600$ mph.:

$$(150)(-450) + (200)(-600) = (250) \cdot \frac{ds}{dt} \rightarrow$$



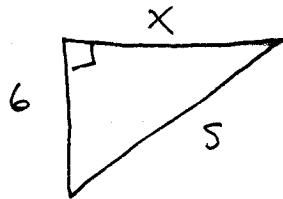
$$\frac{ds}{dt} = -750 \text{ mph.}$$

b.) Assuming that the planes would "meet" at the "corner", we have

$$D = RT \rightarrow 200 = 600 T \rightarrow T = \frac{1}{3} \text{ hr}$$

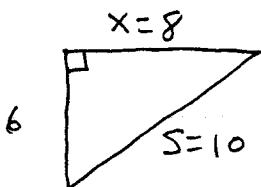
or $T = 20 \text{ min.}$

20.)



$$x^2 + 6^2 = s^2 \rightarrow$$

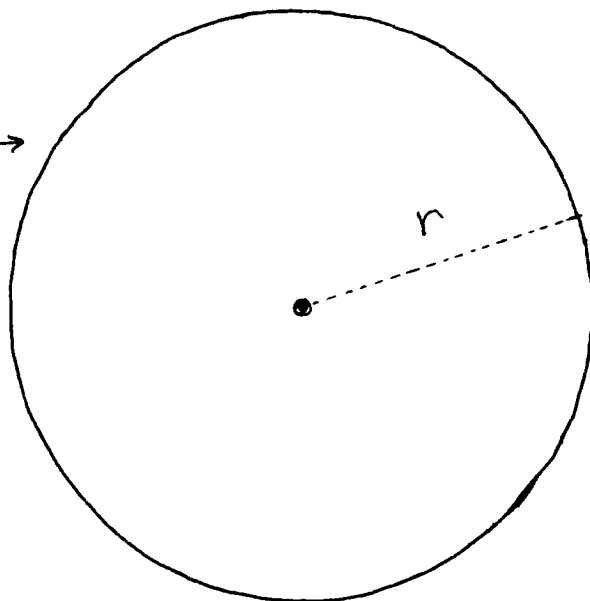
$$\cancel{2x} \cdot \frac{dx}{dt} = \cancel{2s} \cdot \frac{ds}{dt} \rightarrow$$



$$(8) \frac{dx}{dt} = (10)(240) \rightarrow$$

$$\frac{dx}{dt} = 300 \text{ mph.}$$

23.) thickness $\rightarrow$
is 0.08 ft. of
spreading oil
slick; volume
of slick (cylinder)
is



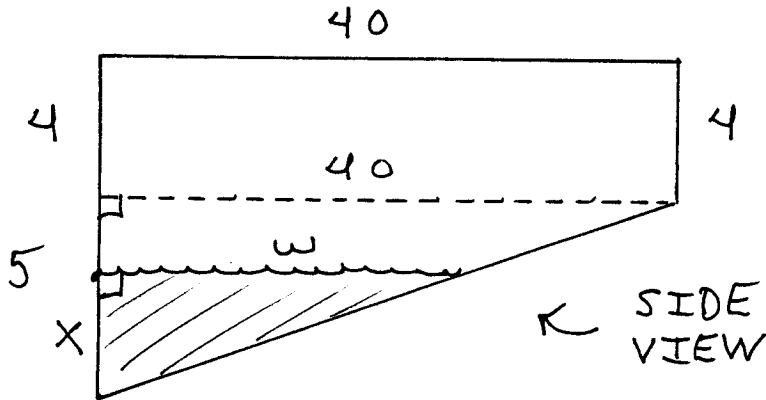
$$V = \pi r^2 h$$
$$= \pi r^2 (0.08) = (0.08)\pi r^2$$

find $\frac{dV}{dt}$ when $r = 750$ ft. and $\frac{dr}{dt} = 0.5$ ft./min.:

$$V = (0.08)\pi r^2 \rightarrow$$
$$\frac{dV}{dt} = (0.08)\pi \cdot 2r \cdot \frac{dr}{dt}$$
$$= (0.08)\pi \cdot 2(750)(0.5)$$
$$= 60\pi \text{ ft.}^3/\text{min.}$$

Chapter 2 Review

99.) $\frac{dV}{dt} = 10 \text{ ft}^3/\text{min.}$, find $\frac{dx}{dt}$
when $x = 4 \text{ ft.}$:



by similar triangles

$$\frac{40}{w} = \frac{5}{x}$$

$\rightarrow 5w = 40x \rightarrow \boxed{w = 8x}$; the volume of the water in the pool is

$$\begin{aligned} V &= (\text{area of } \triangle)(20) \\ &= \frac{1}{2}(\text{base})(\text{height})(20) \\ &= 10(x)(w) \\ &= 10(x)(8x) \quad \text{or} \quad \boxed{V = 80x^2}; \end{aligned}$$

$$\frac{d}{dt} \rightarrow \frac{dV}{dt} = 80 \cdot 2x \cdot \frac{dx}{dt}$$

$$\rightarrow 10 = 80 \cdot 2(4) \cdot \frac{dx}{dt}$$

$$\rightarrow \frac{dx}{dt} = \frac{10}{640} = \boxed{\frac{1}{64} \text{ ft./min.}}$$

Math 16A
Kouba
Worksheet 7

1.) A woman 6 feet tall is walking away from a street lamp which is 10 feet high at the rate of 3 ft./sec. How fast is the tip of her shadow moving away from her feet when she is

- a.) 10 feet from the base of the street lamp ?
- b.) 100 feet from the base of the street lamp ?

2.) A small helium-filled balloon sits 20 feet from the base of a street lamp which is 10 feet high. The balloon is released and rises vertically at the constant rate of 2 ft./sec. How fast is the tip of the balloon's shadow moving away from the base of the street lamp when the balloon is

- a.) 4 feet above the ground ?
- b.) 8 feet above the ground ?
- c.) 9.5 feet above the ground ?

3.) The volume of a cube is changing at the constant rate of $15 \text{ ft.}^3/\text{min.}$ At what rate is the surface area of the cube changing when

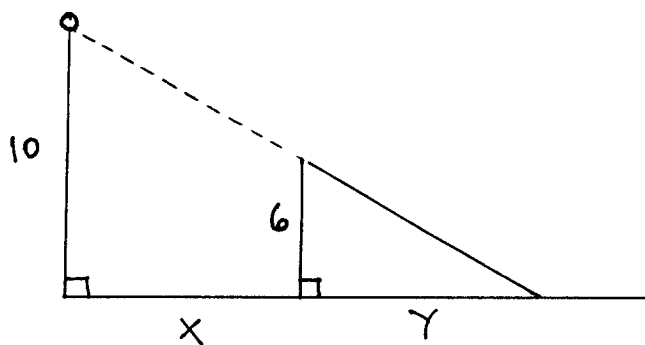
- a.) the surface area is 24 ft.^2 ?
- b.) the surface area is 150 ft.^2 ?

Worksheet 7

1.) $\frac{dx}{dt} = 3 \text{ ft./sec.}$

$$\frac{10}{x+y} = \frac{6}{y} \rightarrow$$

$$10y = 6x + 6y \rightarrow 4y = 6x \rightarrow \boxed{y = \frac{3}{2}x} ;$$



a.) Find $\frac{dy}{dt}$ when $x = 10 \text{ ft.}$:

$$\frac{dy}{dt} = \frac{3}{2} \cdot \frac{dx}{dt} = \frac{3}{2} \cdot 3 = \frac{9}{2} \text{ ft./sec.}$$

b.) Find $\frac{dy}{dt}$ when $x = 100 \text{ ft.}$:

$$\frac{dy}{dt} = \frac{3}{2} \cdot \frac{dx}{dt} = \frac{3}{2} \cdot 3 = \frac{9}{2} \text{ ft./sec.}$$

2.) $\frac{dx}{dt} = 2 \text{ ft./sec.}$

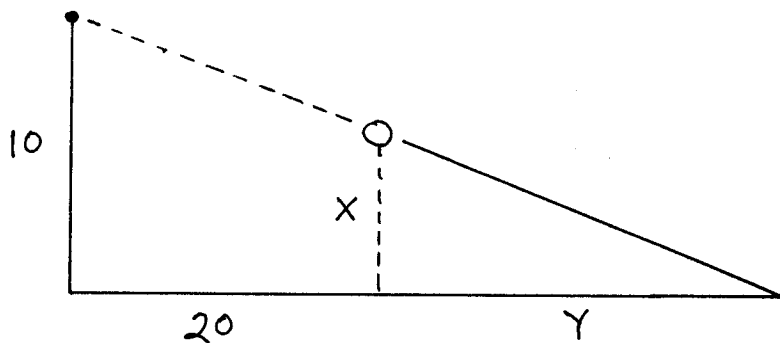
$$\frac{10}{20+y} = \frac{x}{y} \rightarrow$$

$$10y = 20x + xy \rightarrow$$

$$10 \frac{dy}{dt} = 20 \frac{dx}{dt} + x \cdot \frac{dy}{dt} + \frac{dx}{dt} \cdot y \rightarrow$$

$$(10-x) \frac{dy}{dt} = (20+y) \frac{dx}{dt} \rightarrow$$

$$\frac{dy}{dt} = \frac{20+y}{10-x} \cdot \frac{dx}{dt} ;$$



a.) Find $\frac{dY}{dt}$ when $X = 4$ ft. :

$$10Y = 20X + XY \rightarrow 10Y = 20(4) + (4)Y \rightarrow$$

$$6Y = 80 \rightarrow Y = \frac{40}{3} \text{ ft. so}$$

$$\frac{dY}{dt} = \frac{20 + \frac{40}{3}}{10 - 4} \cdot (2) = \left(\frac{100}{9} \text{ ft./sec.} \right)$$

b.) Find $\frac{dY}{dt}$ when $X = 8$ ft. :

$$10Y = 20X + XY \rightarrow 10Y = 20(8) + (8)Y$$

$$\rightarrow 2Y = 160 \rightarrow Y = 80 \text{ ft. so}$$

$$\frac{dY}{dt} = \frac{20 + 80}{10 - 8} \cdot (2) = \left(100 \text{ ft./sec.} \right)$$

c.) Find $\frac{dY}{dt}$ when $X = 9\frac{1}{2}$ ft. :

$$10Y = 20X + XY \rightarrow 10Y = 20\left(\frac{19}{2}\right) + \left(\frac{19}{2}\right)Y \rightarrow$$

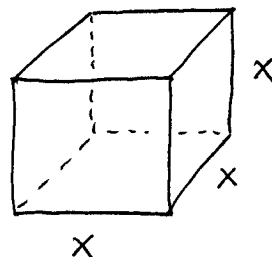
$$\frac{1}{2}Y = 190 \rightarrow Y = 380 \text{ ft. so}$$

$$\frac{dY}{dt} = \frac{20 + 380}{10 - 9\frac{1}{2}} (2) = \left(1600 \text{ ft./sec.} \right)$$

$$3.) \frac{dV}{dt} = 15 \text{ ft}^3/\text{min}.$$

where volume

$$V = x^3 \rightarrow$$



$$\frac{dV}{dt} = 3x^2 \cdot \frac{dx}{dt} \rightarrow 15 = 3x^2 \frac{dx}{dt}$$

$$\rightarrow \boxed{\frac{dx}{dt} = \frac{5}{x^2}} ;$$

surface area $S = 6x^2 \rightarrow$

$$\boxed{\frac{dS}{dt} = 12x \cdot \frac{dx}{dt} = 12x \cdot \frac{5}{x^2} = \frac{60}{x}} ;$$

a.) Find $\frac{dS}{dt}$ when $S = 24$:

$$S = 24 \rightarrow 6x^2 = 24 \rightarrow x = 2 \text{ ft. so}$$

$$\frac{dS}{dt} = \frac{60}{2} = \boxed{30 \text{ ft}^2/\text{min}.}$$

b.) Find $\frac{dS}{dt}$ when $S = 150$:

$$S = 150 \rightarrow 6x^2 = 150 \rightarrow x = 5 \text{ ft. so}$$

$$\frac{dS}{dt} = \frac{60}{5} = \boxed{12 \text{ ft}^2/\text{min}.}$$