Section 1.5

2.) $f(x) = x^2 - 3x + 1$

| $x$ | $f(x)$ |
|---|---|
| 1.9 | −1.09 |
| 1.99 | −1.0099 |
| 1.999 | −1.000999 |
| 2.001 | −0.998999 |
| 2.01 | −0.9899 |
| 2.1 | −0.89 |

Guess: $\lim\limits_{x \to 2} (x^2 - 3x + 1) = -1$

6.) $f(x) = \dfrac{\sqrt{x+2} - \sqrt{2}}{x}$

| $x$ | $f(x)$ |
|---|---|
| −0.1 | 0.358086 |
| −0.01 | 0.353996 |
| −0.001 | 0.353597 |
| 0.001 | 0.353509 |
| 0.01 | 0.353112 |
| 0.1 | 0.349241 |

Guess: $\lim\limits_{x \to 0} \dfrac{\sqrt{x+2} - \sqrt{2}}{x} \approx 0.3535$

9.) a.) $\lim\limits_{x \to 0} f(x) = 1$  b.) $\lim\limits_{x \to -1} f(x) = 3$

10.) a.) $\lim\limits_{x \to 1} f(x) = -2$  b.) $\lim\limits_{x \to 3} f(x) = 0$

11.) a.) $\lim\limits_{x \to 0} g(x) = 1$  b.) $\lim\limits_{x \to -1} g(x) = 3$

12.) a.) $\lim\limits_{x \to -2} h(x) = -5$  b.) $\lim\limits_{x \to 0} h(x) = -3$

17.) a.) $\lim\limits_{x \to 3^+} f(x) = 1$

b.) $\lim\limits_{x \to 3^-} f(x) = 1$

c.) $\lim\limits_{x \to 3} f(x) = 1$

19.) a.) $\lim\limits_{x \to 3^+} f(x) = 0$

b.) $\lim\limits_{x \to 3^-} f(x) = 0$

c.) $\lim\limits_{x \to 3} f(x) = 0$

22.) a.) $\lim_{x \to -1^+} f(x) = 0$

b.) $\lim_{x \to -1^-} f(x) = 2$

c.) $\lim_{x \to -1} f(x)$ DNE

23.) $\lim_{x \to 2} x^4 = 2^4 = 16$

29.) $\lim_{x \to 3} \sqrt{x+1} = \sqrt{4} = 2$

32.) $\lim_{x \to -2} \dfrac{3x+1}{2-x} = \dfrac{-5}{4}$

37.) $\lim_{x \to 3} \dfrac{\sqrt{x+1}-1}{x} = \dfrac{2-1}{3} = \dfrac{1}{3}$

40.) $\lim_{x \to 2} \dfrac{\frac{1}{x+2} - \frac{1}{2}}{x} = \dfrac{\frac{1}{4} - \frac{1}{2}}{2} = \dfrac{\frac{-1}{4}}{\frac{2}{1}} = \dfrac{-1}{8}$

42.) $\lim_{x \to -1} \dfrac{2x^2-x-3}{x+1} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x \to -1} \dfrac{(2x-3)\cancel{(x+1)}}{\cancel{x+1}}$

$= \lim_{x \to -1} (2x-3) = -5$

44.) $\lim_{x \to 2} \dfrac{2-x}{x^2-4} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x \to 2} \dfrac{-\cancel{(x-2)}}{\cancel{(x-2)}(x+2)} = \dfrac{-1}{4}$

47.) $\lim_{x \to -2} \dfrac{x^3+8}{x+2} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x \to -2} \dfrac{\cancel{(x+2)}(x^2-2x+4)}{\cancel{x+2}}$

$= 4+4+4 = 12$

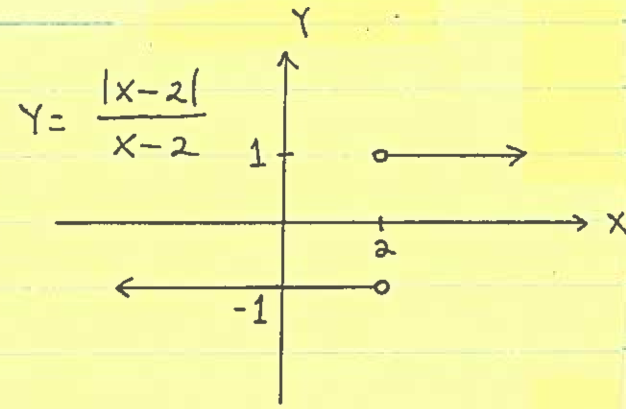
50.) since

$$\lim_{x\to 2^+} \frac{|x-2|}{x-2} = \lim_{x\to 2^+} 1 = 1$$

and

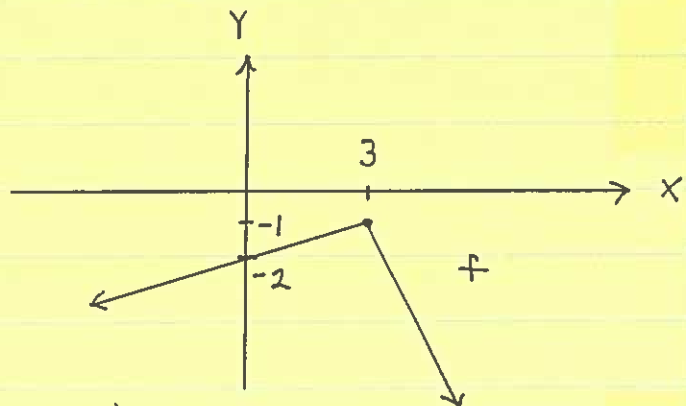
$$\lim_{x\to 2^-} \frac{|x-2|}{x-2} = \lim_{x\to 2^-} -1 = -1$$

so $\lim_{x\to 2} \frac{|x-2|}{x-2}$ does not exist.




51.)

$$f(x) = \begin{cases} \frac{1}{3}x - 2, & x \le 3 \\ -2x+5, & x > 3 \end{cases}$$




$$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (-2x+5) = -1$$

and $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} \left(\frac{1}{3}x - 2\right) = -1$

so $\lim_{x\to 3} f(x) = -1$.

52.) $f(s) = \begin{cases} s, & s \le 1 \\ 1-s, & s > 1 \end{cases}$ so

$\lim_{s\to 1^+} f(s) = \lim_{s\to 1^+} (1-s) = 1-(1) = 0$ and

$\lim_{s\to 1^-} f(s) = \lim_{s\to 1^-} s = 1$, so $\lim_{s\to 1} f(s)$ does not exist.

56.) $$\lim_{\Delta x\to 0} \frac{\sqrt{x+\Delta x} - \sqrt{x}}{\Delta x} = \text{"}\frac{0}{0}\text{"}$$

$$= \lim_{\Delta x\to 0} \frac{\sqrt{x+\Delta x} - \sqrt{x}}{\Delta x} \cdot \frac{\sqrt{x+\Delta x} + \sqrt{x}}{\sqrt{x+\Delta x} + \sqrt{x}}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{(x}+\Delta x) - \cancel{x}}{\Delta x\left(\sqrt{x+\Delta x} + \sqrt{x}\right)} = \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}}{\cancel{\Delta x}\left(\sqrt{x+\Delta x} + \sqrt{x}\right)}$$

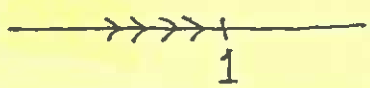
$$= \frac{1}{\sqrt{x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

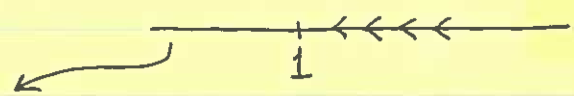
57.) $$\lim_{\Delta t \to 0} \frac{(t+\Delta t)^2 - 5(t+\Delta t) - (t^2 - 5t)}{\Delta t}$$

$$\overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{\Delta t \to 0} \frac{\cancel{t^2} + 2t\cdot\Delta t + (\Delta t)^2 \cancel{- 5t} - 5\cdot\Delta t \cancel{- t^2} \cancel{+ 5t}}{\Delta t}$$

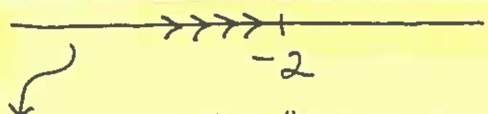
$$= \lim_{\Delta t \to 0} \frac{\Delta t\cdot(2t + \Delta t - 5)}{\Delta t} = 2t + 0 - 5 = 2t - 5$$

59.) $$\lim_{x \to 1^-} \frac{2}{x^2 - 1} = \text{"}\frac{2}{0^-}\text{"} = -\infty$$



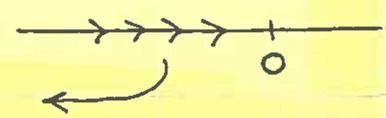


60.) $$\lim_{x \to 1^+} \frac{5}{1-x} = \text{"}\frac{5}{0^-}\text{"} = -\infty$$



61.) $$\lim_{x \to -2^-} \frac{1}{x+2} = \text{"}\frac{1}{0^-}\text{"} = -\infty$$

62.) $$\lim_{x \to 0^-} \frac{x+1}{x} = \text{"}\frac{1}{0^-}\text{"} = -\infty$$

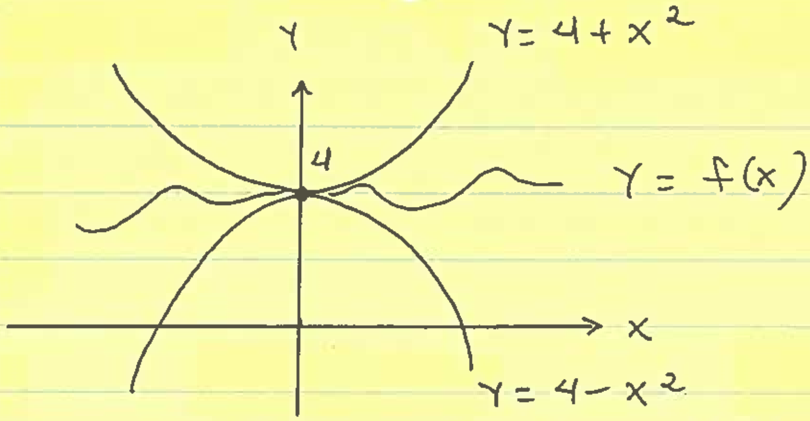


64.) $\lim\limits_{x \to 1} \dfrac{x^2+6x-7}{x^3-x^2+2x-2} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim\limits_{x \to 1} \dfrac{\cancel{(x-1)}(x+7)}{\cancel{(x-1)}(x^2+2)}$

$$\begin{array}{r} x^2 + 2 \\ x-1 \overline{\smash{)}\,x^3 - x^2 + 2x - 2} \\ \underline{x^3 - x^2} \\ 2x - 2 \\ \underline{2x - 2} \end{array}$$

$= \lim\limits_{x \to 1} \dfrac{x+7}{x^2+2} = \dfrac{8}{3}$

68.)




Since $4 - x^2 \le f(x) \le 4 + x^2$ it follows that

$$\lim_{x \to 0} (4 - x^2) \le \lim_{x \to 0} f(x) \le \lim_{x \to 0} (4 + x^2)$$

or

$$4 \le \lim_{x \to 0} f(x) \le 4 \quad ;$$

thus $\lim\limits_{x \to 0} f(x) = 4$ .

Worksheet 1

11.) a.) $Y = \dfrac{2-3X}{X-1} \to Y(X-1) = 2-3X \to$

$XY - Y = 2 - 3X \to XY + 3X = Y + 2 \to$

$X(Y+3) = Y+2 \to X = \dfrac{Y+2}{Y+3} = f^{-1}(Y)$

$\left(\text{or } Y = \dfrac{X+2}{X+3} = f^{-1}(X)\right)$

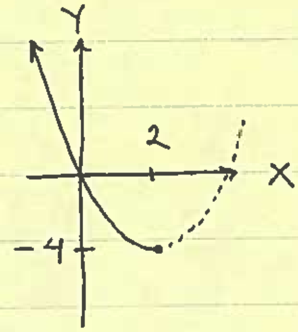
b.) $Y = X^2 - 4X$ for $X \le 2 \to$

$Y = (X^2 - 4X + 4) - 4$ for $X \le 2 \to$

$Y = (X-2)^2 - 4 \to Y + 4 = (X-2)^2 \to$

$X - 2 = \pm\sqrt{Y+4} \to X = 2 \pm \sqrt{Y+4} \to$

why? $X = 2 - \sqrt{Y+4} = f^{-1}(Y)$




$\left(\text{or } Y = 2 - \sqrt{X+4} = f^{-1}(X)\right)$

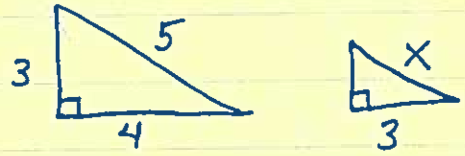
12.) $f(g(x)) = \dfrac{g(x)}{g(x)+5} = 5 - x^3 \to g(x) = (5-x^3)(g(x)+5) \to$

$g(x) = 5g(x) - x^3 g(x) + 25 - 5x^3 \to$

$x^3 g(x) - 4g(x) = 25 - 5x^3 \to (x^3-4)g(x) = 25 - 5x^3 \to$

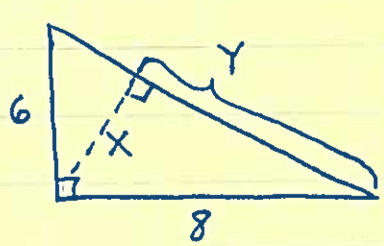
$g(x) = \dfrac{25-5x^3}{x^3-4}$ .

14.) a.) Use similar triangles:

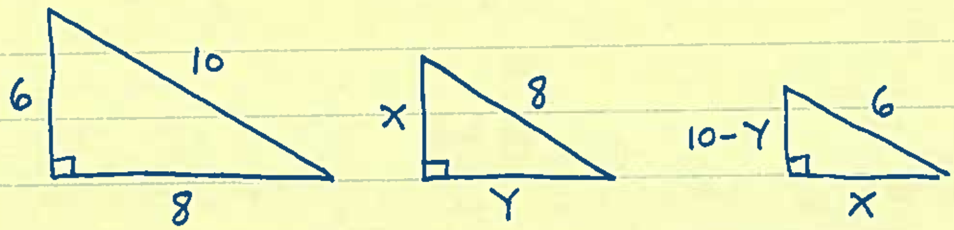



$\dfrac{5}{4} = \dfrac{X}{3} \to X = \dfrac{15}{4}$

b.)




Use similar triangles:



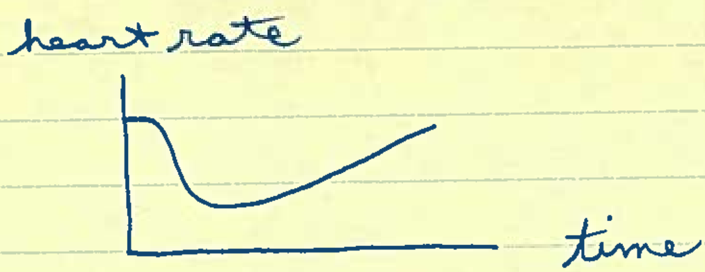

$\frac{6}{8} = \frac{X}{Y} \rightarrow Y = \frac{4}{3}X$ and

$\frac{6}{8} = \frac{10-Y}{X} \rightarrow \frac{3}{4}X = 10 - Y \rightarrow \frac{3}{4}X = 10 - \frac{4}{3}X \rightarrow$

$\left(\frac{3}{4} + \frac{4}{3}\right)X = 10 \rightarrow \frac{25}{12}X = 10 \rightarrow X = \frac{24}{5}$

15.) a.) IV b.) II c.) III

16.)




19.) domain : $-1 \leq X \leq 1.5$

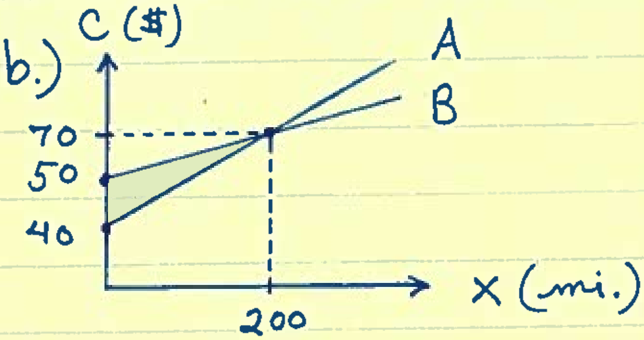
range : $-1.5 \leq Y \leq 0$

23.) Let X : miles traveled

a.) Co. A : cost $C = 40 + 0.15X = A(x)$

Co. B : cost $C = 50 + 0.1X = B(x)$

b.)




Find pt. of intersection :

$40 + 0.15X = 50 + 0.1X \rightarrow$

$0.05X = 10 \rightarrow$

$X = 200$ mi.

c.) Use Co. A if $X \leq 200$ miles ;
use Co. B if $X > 200$ miles .

SA2: Volume $V = \frac{4}{3}\cancel{\pi} r^3 = \frac{\cancel{\pi}}{6} \rightarrow r^3 = \frac{1}{8} \rightarrow$

$r = \frac{1}{2}$ ft. so diameter of balls is $d = 1$ ft.

a.) The number of balls will be

$6 \times 4 \times 3 = 72$ balls

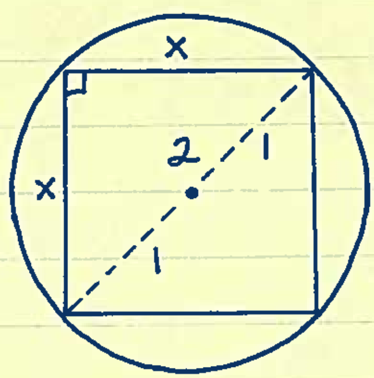
b.) The available space in box is

(Volume of box) − (Volume of balls)

$= 72 - 72\left(\frac{\pi}{6}\right) \approx 34.3\ \text{ft.}^3$, so total weight

of $H_2O$ is $W = (34.3)(62.5) \approx 2143.81$ lbs.

SA5:




Circumference

$C = 2\pi r = 2\pi \rightarrow r = 1$ ft.,

by Pythagorean Theorem

$x^2 + x^2 = 2^2 \rightarrow$

$2x^2 = 4 \rightarrow x = \sqrt{2}$ ft,

so perimeter of square is

$P = 4\sqrt{2}$ ft.