Ex: Assume that the number of elk in an isolated herd in Yellowstone National Park at time \( t \) (years) is given by

\[
N(t) = 400 - \frac{4500}{t^2 + 30}
\]

for \( t \geq 0 \).

Note that \( N(0) = 250 \) so initially there are 250 elk in the herd. Since

\[
\lim_{t \to +\infty} N(t) = 400
\]

we would expect that after a "long period of time" the number of elk in the herd would stabilize at 400. Differentiation gives

\[
N'(t) = 4500 (t^2 + 30)^{-2} (2t) = \frac{900t}{(t^2 + 30)^2}
\]

\[
\frac{1}{10} + +
\]

\[
N' \quad \text{since the units of } N'(t)
\]

\[
\text{are elk per year}
\]

\[
\text{the following examples show the rate at which the herd is increasing at specific times:}
\]
\[ N'(t) = 9.4 \text{ elk per year}, \]
\[ N'(5) = 14.9 \text{ elk per year}, \]
\[ N'(10) = 5.3 \text{ elk per year}. \]

Differentiating again,
\[ N''(t) = \frac{9000 [30 - 3t^2]}{(t^2 + 30)^2}. \]

What is the biological significance of this inflection point? It represents the time at which the size of the elk herd is increasing most rapidly. The graph of \( N(t) \) follows.

\[ N \text{ (elk)} \]

\[ \begin{align*}
N''(t) &+ 0 - \\
t = 0 &\quad t = t_0 \approx 3.16 \text{ years} \\
N &\approx 287 \text{ elk} \\
N' &\approx 17.8 \text{ elk/yr.}
\end{align*} \]

Elk herd increasing at maximum rate of 17.8 elk/yr.

\[ t = t_0 \approx 3.16 \text{ yrs.} \]