Math 16A
Section 2.2

Rules for Differentiation

RECALL: I.) If $y = f(x)$, then the derivative $f'(x)$ is the SLOPE of the line tangent to the graph of $y = f(x)$ at x.

II.) Notation:

$$f'(x) = y' = Df(x) = \frac{dy}{dx}$$

Derivative Rules (Shortcuts):

1.) If $f(x) = c$, a constant, then $f'(x) = 0$.

2.) If $f(x) = mx + b$, then $f'(x) = m$.

3.) If $f(x) = x^n$, where n is any #, then $f'(x) = nx^{n-1}$.

4.) $D(f(x) \pm g(x)) = f'(x) \pm g'(x)$

5.) $D(c \cdot f(x)) = c \cdot f'(x)$

Verify Rule 3 for $n = 3$:

$$f(x) = x^3 \xrightarrow{D} f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 - \cancel{x^3}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(3x^2 + 3xh + h^2)}{\cancel{h}}$$

$$= 3x^2 + 3x(0) + (0)^2 = 3x^2, \text{ i.e.,}$$

$$D(x^3) = 3x^2 .$$

Example: Use Rules 1–5 to differentiate each function. Do not simplify answers.

1.) $y = x^3 + x^{-2} - 3 \xrightarrow{D}$

$y' = 3x^2 + {}^-2x^{-3} - 0$

2.) $y = 5x^{1/3} - 9x + 7^5 \xrightarrow{D}$

$y' = 5 \cdot \frac{1}{3} x^{-2/3} - 9 + 0$

3.) $f(x) = \dfrac{x}{7} + \dfrac{7}{x} \longrightarrow$

$f(x) = \frac{1}{7}x + 7x^{-1} \xrightarrow{D} f'(x) = \frac{1}{7} + 7(-1)x^{-2}$

4.) $y = (x^2 - 1)(2x + 3) \longrightarrow$

$y = 2x^3 + 3x^2 - 2x - 3 \xrightarrow{D}$

$y' = 6x^2 + 6x - 2 - 0$

5.) $g(x) = \dfrac{(3x-1)^2}{x^2} \longrightarrow$

$$g(x) = \frac{9x^2 - 6x + 1}{x^2}$$

$$= \frac{9x^2}{x^2} - \frac{6x}{x^2} + \frac{1}{x^2}$$

$$= 9 - 6x^{-1} + x^{-2} \xrightarrow{D}$$

$$g'(x) = 0 + 6x^{-2} - 2x^{-3} .$$

<u>Example</u>: Solve $f'(x) = 0$ for x and set up a Sign Chart for f'.

1.) $y = \dfrac{x^4}{4} + \dfrac{x^3}{3} - x^2 \longrightarrow$

$y = \frac{1}{4}x^4 + \frac{1}{3}x^3 - x^2 \xrightarrow{D}$

$$y' = \frac{1}{4}\cdot 4x^3 + \frac{1}{3}\cdot 3x^2 - 2x$$
$$= x^3 + x^2 - 2x$$
$$= x\,(x^2 + x - 2)$$
$$= x\,(x-1)(x+2) = 0 \rightarrow x = 0, 1, -2$$

$$\begin{array}{ccccccccc} & - & 0 & + & 0 & - & 0 & + & \\ \hline & -3 & x=-2 & -1 & x=0 & 1/2 & x=1 & 2 & y' \end{array}$$

2.) $$y = \frac{x^3+16}{x} \longrightarrow$$

$$y = \frac{x^3}{x} + \frac{16}{x} = x^2 + 16x^{-1} \xrightarrow{D}$$

$$y' = 2x - 16x^{-2} = \frac{2x}{1} - \frac{16}{x^2}$$
$$= \frac{2x^3 - 16}{x^2} = 0 \longrightarrow$$

$$2x^3 - 16 = 2\,(x^3 - 8) = 0 \rightarrow x = 2$$

$$\begin{array}{cccccccc} & & \text{NO} & & & & & \\ & - & & - & 0 & + & & \\ \hline & -1 & x=0 & 1 & x=2 & 3 & & y' \end{array}$$

Example : assume that the weight (lbs.) of a calf is given by

$$w(t) = t^3 - 3t^2 + 35 ,$$

where $t \geq 0$ is given in months.

1.) What is the calf's weight when $t = 0, 1,$ and 4 ?

2.) At what RATE is the calf's weight changing when $t = 1, 2,$ and 3 ?

1.) $w(0) = (0)^3 - 3(0)^2 + 35 = 35$ lbs. ;

$w(1) = (1)^3 - 3(1)^2 + 35 = 33$ lbs. ;

$w(4) = (4)^3 - 3(4)^2 + 35 = 51$ lbs.

2.) $\xrightarrow{D} w'(t) = 3t^2 - 6t$;

$w'(1) = 3(1)^2 - 6(1) = -3$ lbs./mo. ;

$w'(2) = 3(2)^2 - 6(2) = 0$ lbs./mo. ;

$w'(3) = 3(3)^2 - 6(3) = 9$ lbs./mo.