$\boxed{\text{Math 16A} \quad \text{Section 3.4}}$

## Applied Extrema (Max./Min.) Problems

Example: The sum of two positive numbers is 48. What should the numbers be so that the SUM of the cube of one # and the square of the other # is a MINIMUM?

assume that $x, y$ are (+) #'s with

$$x + y = 48 \rightarrow \boxed{y = 48 - x} \; ;$$

minimize the Sum

$$S = x^3 + y^2 = x^3 + (4-x)^2 \rightarrow$$

$$\boxed{S = x^3 + (48 - x)^2} \xrightarrow{D}$$

$$S' = 3x^2 + 2(48 - x)(-1) = 0 \rightarrow$$

$$3x^2 + 2x - 96 = 0 \rightarrow$$

$$(3x-16)(x+6) = 0 \rightarrow$$

$$x = -6 \text{ (NO!) or } x = \frac{16}{3} \rightarrow$$

$-\quad 0 \quad +$ $\quad S'$

$$x = 16/3 = 5\tfrac{1}{3},$$

$$y = 42\tfrac{2}{3}, \text{ and}$$

Minimum sum is

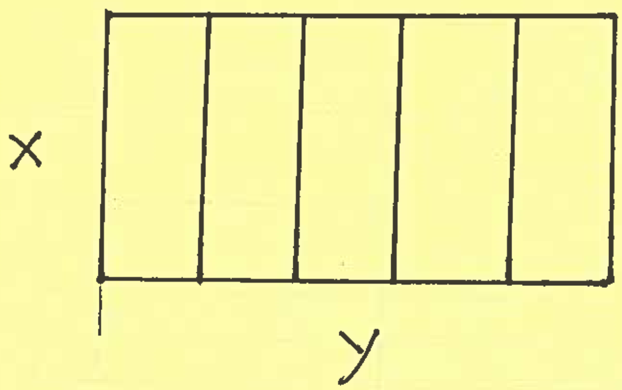
$$S = x^3 + y^2 = \left(\frac{16}{3}\right)^3 + \left(\frac{128}{3}\right)^2$$

$$= \frac{4096}{27} + \frac{16,384}{9}$$

$$= \frac{4096}{27} + \frac{49,152}{27}, \text{ i.e.,}$$

$$S = \frac{53,248}{27}.$$

Example: A rectangular pen with four partitions (dividers) is to be constructed using 1200 ft. of fencing. What dimensions of the pen will MAXIMIZE the total AREA of the pen?




Total fencing is

$$1200 = 6X + 2Y \rightarrow$$

$$600 = 3X + y \rightarrow$$

$$\boxed{y = 600 - 3X} \;;$$

Maximize the total Area

$$A = XY = X(600 - 3X) \rightarrow$$

$$\boxed{A = 600X - 3X^2} \xrightarrow{D}$$

$$A' = 600 - 6X = 0 \rightarrow$$

$$X = 100 \text{ ft.}$$

$$\underset{\phantom{x}}{+ \quad 0 \quad -} \quad A'$$

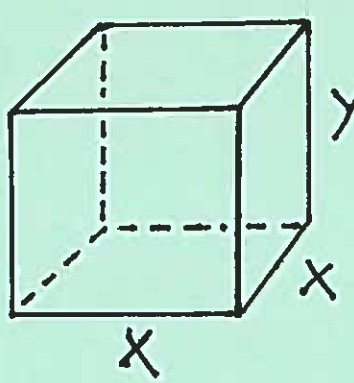
$X = 100$ ft.,

$Y = 300$ ft., and

Maximum Area is

$$A = XY = (100)(300) = 30{,}000 \text{ ft.}^2$$

__Example__: You are to construct a closed rectangular box with a square base. Material for the __top__ and __bottom__ costs $\$3/\text{ft.}^2$ and material for the __sides__ costs $\$2/\text{ft.}^2$ What dimensions of the box will MINIMIZE construction costs if the Volume of the box must be $96$ ft.$^3$ ?




Volume is

$$96 = x^2 y \longrightarrow$$

$$\boxed{y = \frac{96}{x^2}} \; ;$$

Minimize the Total Cost

$$C = C_{top} + C_{bottom} + C_{sides}$$

$$= \left(\$3/ft.^2\right)\left(x^2 ft.^2\right) + \left(\$3/ft.^2\right)\left(x^2 ft.^2\right) + \left(\$2/ft.^2\right)\left(4xy\ ft.^2\right)$$

$$= 3x^2 + 3x^2 + 8xy \quad (\$) \longrightarrow$$

$$C = 6x^2 + 8xy \longrightarrow$$

$$C = 6x^2 + 8x \cdot \frac{96}{x^2} \longrightarrow$$

$$\boxed{C = 6x^2 + \frac{768}{x}} \quad (\$) \xrightarrow{D}$$

$$C' = 12x - \frac{768}{x^2} = \frac{12x^3}{x^2} - \frac{768}{x^2}$$

$$= \frac{12x^3 - 768}{x^2} = \frac{12(x^3 - 64)}{x^2} = 0$$

$$\longrightarrow \quad x^3 - 64 = 0 \longrightarrow x = 4$$

$$\begin{array}{ccc|ccc} & - & & 0 & + & \\ \hline & & & x=4 & & \end{array} \quad C'$$

$x = 4$ ft.,

$y = 6$ ft., and

minimum cost is

$$C = 6(4)^2 + \frac{768}{(4)}$$

$$= \$288$$