Math 16A
Kouba
An Example from Economics

You have a small business which sells boxes of greeting cards. Assume that the demand \( x \) for boxes is inversely proportional to the square of the price \( p \) of a box of cards. If you charge $20 per box, 125 boxes are sold. Your initial investment is $750 and the cost to you for each box is $5. Find the price \( p \) and the number of boxes \( x \) which will result in the maximum profit to you.

\[
x = \frac{c}{p^2} \quad \text{and} \quad p = \$20, \quad x = 125 \quad \text{boxes} \quad \Rightarrow \quad 125 = \frac{c}{400} \quad \Rightarrow \quad c = 50,000
\]

\[
x = \frac{50,000}{p^2} \quad \text{on} \quad \text{price} \quad p = \sqrt{\frac{50,000}{x}} \quad \text{so profit}
\]

\[
Pr = (\text{revenue}) - (\text{cost})
= px - (750 + 5x)
= \sqrt{\frac{50,000}{x}} \cdot x - 750 - 5x = \sqrt{50,000} \cdot \sqrt{x} - 750 - 5x
\]

\[
P_r = \sqrt{50,000} \cdot \frac{1}{2\sqrt{x}} - 5 = 0 \quad \Rightarrow \quad x = 500 \quad \text{boxes}
\]

\[
\frac{0}{1000} - \frac{p^1}{x=500 \text{ boxes}} \quad \text{and max. profit is} \quad P_r = \$1750
\]