Math 16B (Winter 2021)
Kouba
Exam 2

Printing and signing your name below is a verification that no other person assisted you in the completion of this Exam.

PRINT your name __________________ SIGN your name KEY

Show clear, organized supporting work for your answers. Correct answers without supporting work may not receive full credit. Use of unapproved shortcuts may not receive full credit. There are 8 pages. You must submit exactly 8 pages to Gradescope.

1.) Determine the following six indefinite integrals.

a.) (7 pts.) $\int x^3(x^{-2} + 3x^{-5})\ dx$

$$= \int (x + 3x^{-2})\,dx$$

$$= \frac{1}{2}x^2 + 3\frac{x^{-1}}{-1} + C$$

b.) (7 pts.) $\int \frac{1}{x(\ln x)^2}\ dx$ (Let $u = \ln x \xrightarrow{D} du = \frac{1}{x}dx$)

$$= \int \frac{1}{u^2}\,du = \int u^{-2}\,du$$

$$= \frac{u^{-1}}{-1} + C = \frac{(\ln x)^{-1}}{-1} + C$$

c.) (7 pts.) $\int \frac{3x^2 - x + 1}{x+1}\ dx$

$$
\begin{array}{r}
3x - 4 \\
x+1 \overline{\smash{)}\, 3x^2 - x + 1} \\
-(3x^2 + 3x) \\
\hline
-4x + 1 \\
-(-4x - 4) \\
\hline
5
\end{array}
$$

$= \int \left[3x - 4 + \frac{5}{x+1}\right] dx$

$= 3 \cdot \frac{1}{2} x^2 - 4x$

$+ 5 \ln|x+1| + C$

d.) (7 pts.) $\int xe^x\ dx$

(Let $u = x$, $dv = e^x dx$

$\rightarrow du = 1\,dx$, $v = e^x$)

$= xe^x - \int e^x dx$

$= xe^x - e^x + C$

e.) (7 pts.) $\displaystyle\int \frac{x+2}{x-1}\,dx$

(Let $u = x - 1 \xrightarrow{D} du = 1\,dx$ and
Back Sub $\rightarrow x = u+1$)

$$= \int \frac{(u+1)+2}{u}\,du = \int \frac{u+3}{u}\,du$$

$$= \int \left[\frac{u}{u} + \frac{3}{u}\right] du = \int \left[1 + 3\cdot\frac{1}{u}\right] du$$

$$= u + 3\ln|u| + C = (x-1) + 3\ln|x-1| + C$$

f.) (7 pts.) $\displaystyle\int \tan^2 x\ \sec^2 x\ dx$

(Let $u = \tan x \xrightarrow{D}$
$du = \sec^2 x\ dx$)

$$= \int u^2\,du = \frac{1}{3}u^3 + C$$

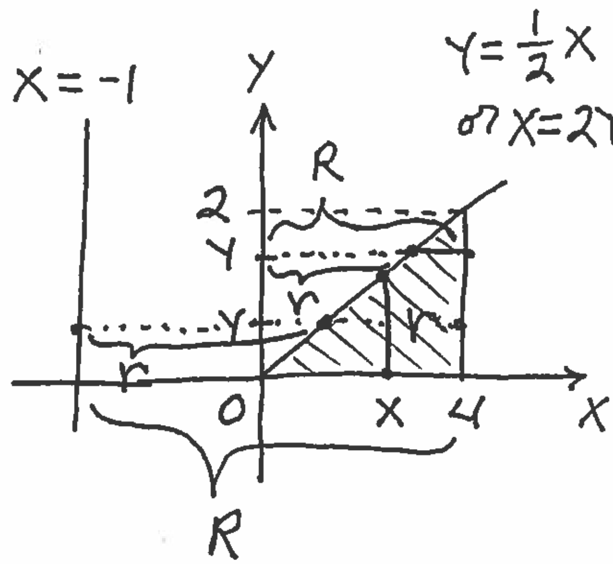
$$= \frac{1}{3}(\tan x)^3 + C$$

2.) Consider the region bounded by the graphs of $y = (1/2)x$, $y = 0$, and $x = 4$. This problem is continued on the next page.

a.) Find the AREA (SET UP ONLY.) of the region using

i.) (6 pts.) vertical cross-sections.

$$AREA = \int_0^4 \frac{1}{2}x \, dx$$



ii.) (6 pts.) horizontal cross-sections.

$$AREA = \int_0^2 (4 - 2Y) \, dy$$

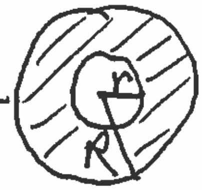
b.) (8 pts.) Find the VOLUME of the solid formed by revolving the region about the x-axis (SET UP ONLY.).

$$VOL = \int_0^4 \pi \left(\frac{1}{2}x\right)^2 dx$$

$A(x) = \pi r^2$ 

c.) (8 pts.) Find the VOLUME of the solid formed by revolving the region about the y-axis (SET UP ONLY.).

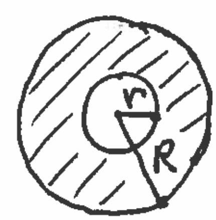
$$A(Y) = \pi R^2 - \pi r^2$$



$$\text{VOL} = \int_0^2 \left[\pi(4)^2 - \pi(2Y)^2\right] dy$$

d.) (8 pts.) Find the VOLUME of the solid formed by revolving the region about the line $x = -1$ (SET UP ONLY.).

$$A(Y) = \pi R^2 - \pi r^2$$



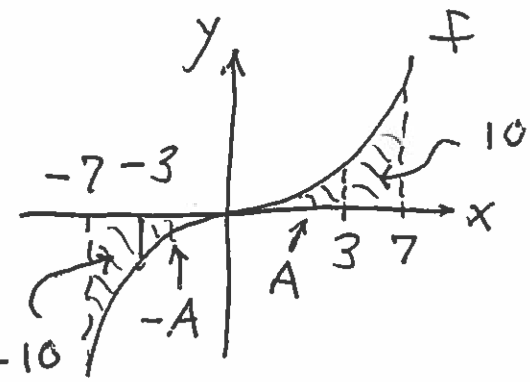
$$\text{VOL} = \int_0^2 \left[\pi(4-(-1))^2 - \pi(2Y-(-1))^2\right] dy$$

3.) (9 pts.) The depth of water in a tank is given by $H = 1 + \frac{2}{\sqrt{t}}$ feet at time t days. Find the AVERAGE depth of the water for $1 \le t \le 4$.

$$AVE = \frac{1}{4-1} \int_1^4 (1 + 2t^{-1/2})\, dt$$
$$= \frac{1}{3}\left[t + 2 \cdot 2t^{1/2}\right]\Big|_1^4$$
$$= \frac{1}{3}(4 + 4\sqrt{4}) - \frac{1}{3}(1 + 4\sqrt{1})$$
$$= \frac{1}{3}(12) - \frac{1}{3}(5) = \frac{7}{3} \text{ ft.}$$

4.) (9 pts.) Assume that f is an ODD function and $\int_{-3}^{7} f(x)\ dx = 10$. Find $\int_{-3}^{-7} f(x)\ dx$.

$$0 = \int_{-7}^{7} f(x)\, dx$$
$$= \int_{-7}^{-3} f(x)\, dx + \int_{-3}^{7} f(x)\, dx$$
$$= -\int_{-3}^{-7} f(x)\, dx + (10)$$
$$\longrightarrow \int_{-3}^{-7} f(x)\, dx = 10$$



5.) (9 pts.) Compute T_4, the Trapezoidal Estimate using $n = 4$, for $\int_{-1}^{3} \sqrt{x^2+1}\, dx$.

$h = \frac{3-(-1)}{4} = 1$

$$T_4 = \frac{h}{2}\left[f(-1) + 2f(0) + 2f(1) + 2f(2) + f(3)\right]$$

$$= \frac{1}{2}\left[\sqrt{2} + 2\sqrt{1} + 2\sqrt{2} + 2\sqrt{5} + \sqrt{10}\right]$$

$$= \frac{1}{2}\left[2 + 3\sqrt{2} + 2\sqrt{5} + \sqrt{10}\right] \approx 6.94$$

6.) (10 pts.) Monsoon rain is falling in Cherrapunji, India, at the rate of $t(t+1)^{1/2}$ inches per day at time t days. Determine the total amount of rainfall for $0 \le t \le 3$.

$$\text{TOTAL} = \int_0^3 t(t+1)^{1/2}\, dt \quad (\text{Let } u = t+1$$

$$\longrightarrow t = u-1 \text{ and } du = dt)$$

$$= \int_{t=0}^{t=3} (u-1)u^{1/2}\, du = \int_{t=0}^{t=3} (u^{3/2} - u^{1/2})\, du$$

$$= \left(\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right)\Big|_{t=0}^{t=3} = \left(\frac{2}{5}(t+1)^{5/2} - \frac{2}{3}(t+1)^{3/2}\right)\Big|_0^3$$

$$= \left(\frac{2}{5}(32) - \frac{2}{3}(8)\right) - \left(\frac{2}{5}(1) - \frac{2}{3}(1)\right)$$

$$= \frac{64}{5} - \frac{16}{3} - \frac{2}{5} + \frac{2}{3} = \frac{62}{5} - \frac{14}{3}$$

$$= \frac{186 - 70}{15} = \frac{116}{15} \approx 7.73 \text{ in.}$$

7.) (10 pts) Integrate $\int \frac{\tan x}{1+\cos x}\, dx$.

$$\int \frac{\tan x}{1+\cos x}\, dx = \int \frac{\tan x \cdot \frac{1}{\cos x}}{(1+\cos x)\cdot \frac{1}{\cos x}}\, dx$$

$$= \int \frac{\sec x \cdot \tan x}{\frac{1}{\cos x} + 1}\, dx = \int \frac{\sec x \tan x}{\sec x + 1}\, dx$$

$$= \ln|\sec x + 1| + C$$