Section 5.5

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2.) $\text{Area} = \int_{-2}^{2}\left[(2x+5)-(x^2+2x+1)\right]dx$

$= \int_{-2}^{2}(-x^2+4)\,dx = \left(\frac{-x^3}{3}+4x\right)\Big|_{-2}^{2} = \left(\frac{-8}{3}+8\right)-\left(\frac{8}{3}-8\right) = \frac{32}{3}$

3.) $\text{Area} = \int_{0}^{3}\left[(-x^2+2x+3)-(x^2-4x+3)\right]dx$

$= \int_{0}^{3}(-2x^2+6x)\,dx = \left(-\frac{2}{3}x^3+3x^2\right)\Big|_{0}^{3} = -18+27 = 9$

4.) $\text{Area} = \int_{0}^{1}(x^2-x^3)\,dx = \left(\frac{x^3}{3}-\frac{x^4}{4}\right)\Big|_{0}^{1} = \frac{1}{3}-\frac{1}{4} = \frac{1}{12}$

6.) $\text{Area} = \int_{0}^{1}\left[(x-1)^3-(x-1)\right]dx + \int_{1}^{2}\left[(x-1)-(x-1)^3\right]dx$

$= 2\int_{0}^{1}\left[(x-1)^3-(x-1)\right]dx = 2\int_{0}^{1}(x^3-3x^2+3x-1-x+1)\,dx$

$= 2\int_{0}^{1}(x^3-3x^2+2x)\,dx = 2\left(\frac{x^4}{4}-x^3+x^2\right)\Big|_{0}^{1}$

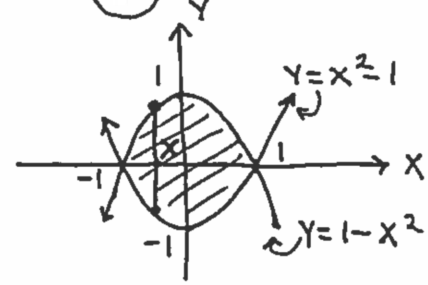
$= 2\left(\frac{1}{4}-1+1\right) = \frac{1}{2}$

8.) $\text{Area} = \int_{1}^{2}\left[(3-x)-\frac{2}{x}\right]dx = \left(3x-\frac{x^2}{2}-2\ln|x|\right)\Big|_{1}^{2}$

$= (6-2-2\ln 2)-\left(3-\frac{1}{2}-2\overset{0}{\cancel{\ln 1}}\right) = \frac{3}{2}-\ln 2^2 = \frac{3}{2}-\ln 4$

10.)

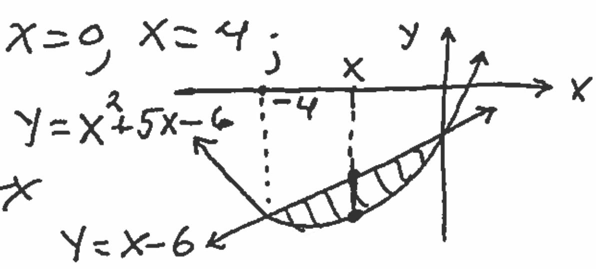
$\text{Area} = \int_{-1}^{1}\left[(1-x^2)-(x^2-1)\right]dx$

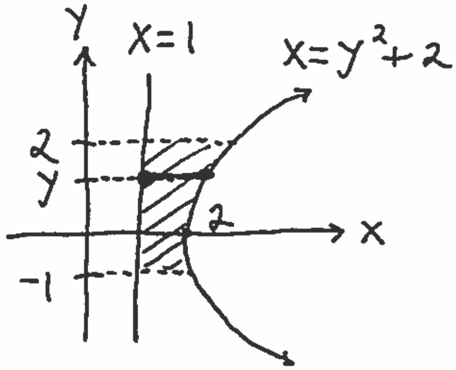


12.) $x-6 = x^2+5x-6 \rightarrow$

$0 = x^2-4x = x(x-4) \rightarrow x=0,\ x=4;$

$\text{Area} = \int_{-4}^{0}\left[(x-6)-(x^2+5x-6)\right]dx$



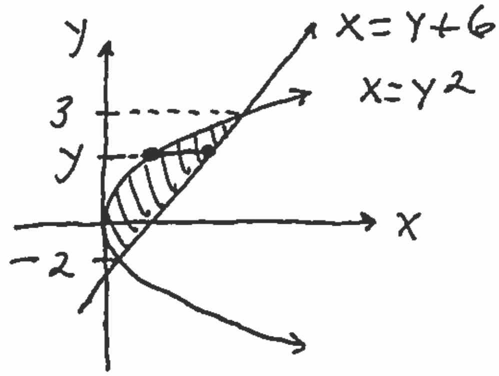
13.) 

$$\text{Area} = \int_{-1}^{2} \left[(y^2+2) - 1 \right] dy$$

14.) $Y + 6 = Y^2 \rightarrow 0 = Y^2 - Y - 6 = (Y-3)(Y+2) \rightarrow$

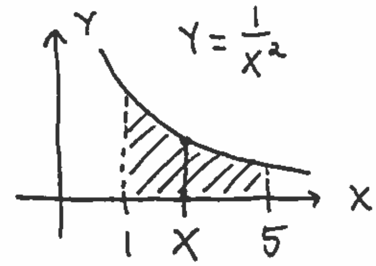
$y = 3, \ y = -2$;

$$\text{Area} = \int_{-2}^{3} \left[(y+6) - y^2 \right] dy$$



15.) $\text{Area} = \int_1^5 \frac{1}{x^2}\, dx = \int_1^5 x^{-2}\, dx$

$= -x^{-1}\Big|_1^5 = \frac{-1}{5} - \frac{-1}{1} = \frac{4}{5}$



17.) intersection : $x^{1/3} = x \rightarrow x = x^3 \rightarrow$

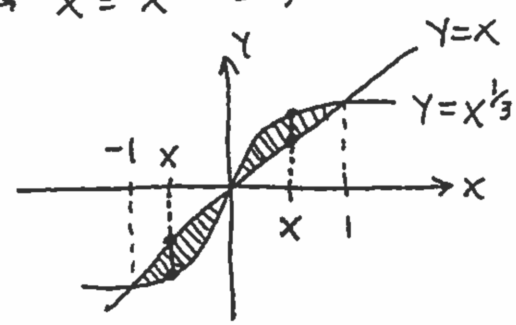
$x - x^3 = x(1-x^2) = x(1-x)(1+x) = 0$

$\rightarrow x = 0, 1, -1$ then

$$\text{Area} = \int_{-1}^{0} (x - x^{1/3})\, dx + \int_0^1 (x^{1/3} - x)\, dx$$

$$= 2\int_0^1 (x^{1/3} - x)\, dx = 2\left(\frac{3}{4} x^{4/3} - \frac{x^2}{2} \right)\Big|_0^1$$

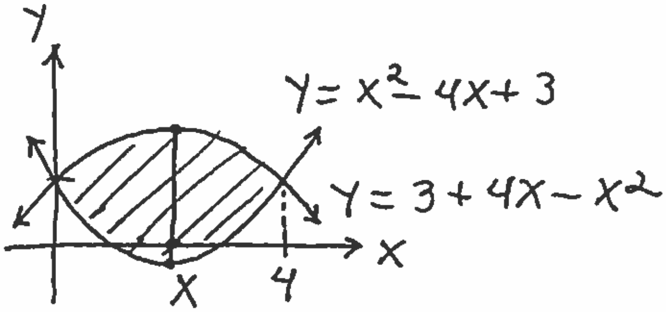
$$= 2\left(\frac{3}{4} - \frac{1}{2} \right) = \frac{1}{2}$$



19.) $X^2 - 4X + 3 = 3 + 4X - X^2 \rightarrow$

$2X^2 - 8X = 0 \rightarrow 2X(X-4) = 0 \rightarrow$

$X = 0, \ X = 4$

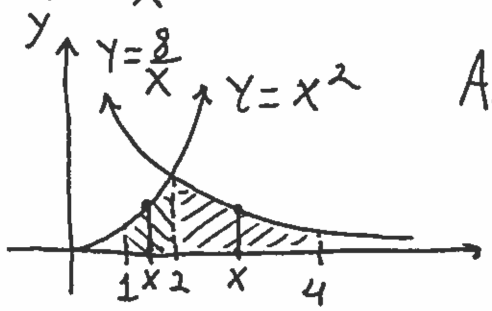


$$\text{Area} = \int_0^4 \left[(3+4x-x^2)-(x^2-4x+3)\right]dx$$

$$= \int_0^4 (8x-2x^2)\,dx = \left(4x^2-\tfrac{2}{3}x^3\right)\Big|_0^4$$

$$= \left(64-\tfrac{2}{3}\cdot 64\right)-(0-0) = \frac{64}{3}$$

23.) $\frac{8}{x} = x^2 \rightarrow x^3 = 8 \rightarrow x = 2$;



$$\text{Area} = \int_1^2 x^2\,dx + \int_2^4 8/x\,dx$$

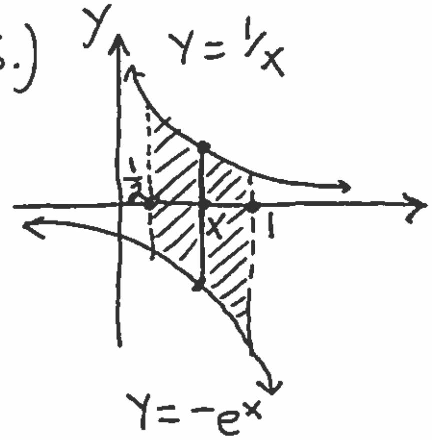
$$= \tfrac{1}{3}x^3\Big|_1^2 + 8\ln|x|\Big|_2^4$$

$$= \left(\tfrac{8}{3}-\tfrac{1}{3}\right) + (8\ln 4 - 8\ln 2)$$

$$= \tfrac{7}{3} + 8\ln 2^2 - 8\ln 2 = \tfrac{7}{3} + 16\ln 2 - 8\ln 2$$

$$= \tfrac{7}{3} + 8\ln 2$$

26.)



$$\text{Area} = \int_{\frac{1}{2}}^{1} \left(\tfrac{1}{x} - (-e^x)\right)dx$$

$$= \int_{\frac{1}{2}}^{1} \left(\tfrac{1}{x} + e^x\right)dx$$

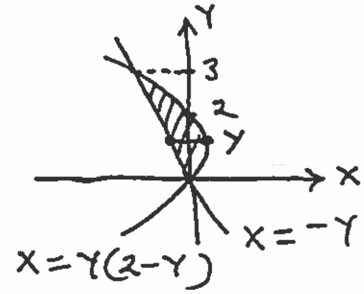
$$= (\ln|x| + e^x)\Big|_{1/2}^{1}$$

$$= (\underbrace{\ln 1}_{0} + e) - (\ln(1/2) + e^{1/2}) = e - \ln(1/2) - e^{1/2}$$

28.) intersection: $Y(2-Y) = -Y \rightarrow 2Y - Y^2 = -Y \rightarrow$
$0 = Y^2 - 3Y = Y(Y-3) \rightarrow Y = 0, 3$ then

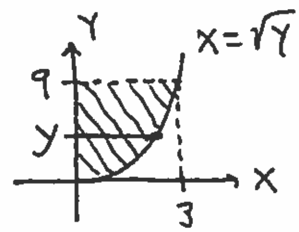
$$\text{Area} = \int_0^3 [Y(2-Y) - (-Y)]\, dy = \int_0^3 (3Y - Y^2)\, dy$$

$$= \left(\frac{3}{2}Y^2 - \frac{Y^3}{3}\right)\Big|_0^3 = \frac{27}{2} - 9 = \frac{9}{2}$$

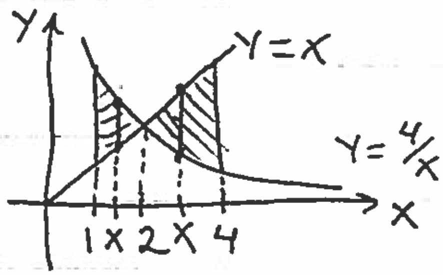


29.) $$\text{Area} = \int_0^9 \sqrt{Y}\, dy = \frac{2}{3} Y^{3/2}\Big|_0^9$$

$$= \frac{2}{3}(9)^{3/2} = \frac{2}{3}(27) = 18$$



33.) $\frac{4}{x} = x \rightarrow x^2 = 4 \rightarrow x = \pm 2 \rightarrow x = 2$;



$$\text{Area} = \int_1^2 \left(\frac{4}{x} - x\right) dx + \int_2^4 \left(x - \frac{4}{x}\right) dx$$

$$= \left(4\ln x - \frac{x^2}{2}\right)\Big|_1^2 + \left(\frac{x^2}{2} - 4\ln x\right)\Big|_2^4$$

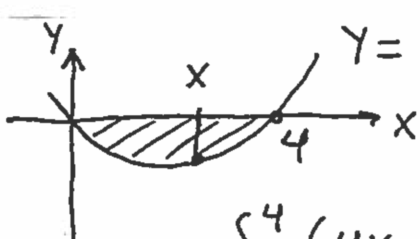
$$= \left((4\ln 2 - 2) - (4\cancel{\ln 1}^{0} - \tfrac{1}{2})\right)$$

$$+ \left((8 - 4\ln 4) - (2 - 4\ln 2)\right) = \cancel{4\ln 2} - \cancel{2} + \frac{1}{2}$$

$$+ 8 - \cancel{4\ln 2^2} - \cancel{2} + \cancel{4\ln 2}$$

$$= 4 + \frac{1}{2} = \frac{9}{2}$$

35.)



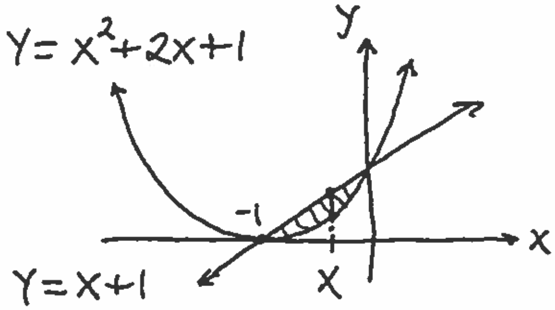
$Y = x^2 - 4x = x(x-4)$

$$\text{Area} = \int_0^4 [0 - (x^2 - 4x)]\, dx$$

$$= \int_0^4 (4x - x^2)\, dx = \left(2x^2 - \frac{1}{3}x^3\right)\Big|_0^4$$

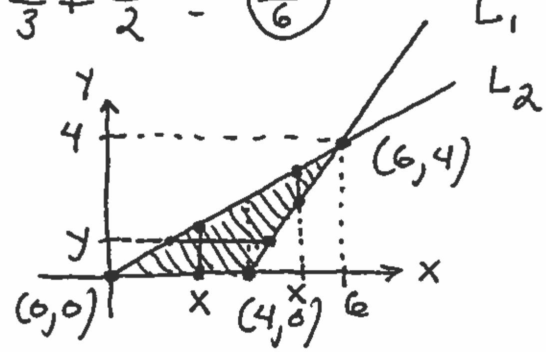
$$= \left(32 - \frac{1}{3}\cdot 64\right) - (0 - 0) = \frac{32}{3}$$

37.) $x^2+2x+1 = x+1 \rightarrow x^2+x=0 \rightarrow$
$x(x+1)=0 \rightarrow x=0,\ x=-1$;



$$\text{Area} = \int_{-1}^{0} \left[(x+1) - (x^2+2x+1)\right] dx$$
$$= \int_{-1}^{0} (-x^2 - x)\, dx$$
$$= \left(-\tfrac{1}{3}x^3 - \tfrac{1}{2}x^2\right)\Big|_{-1}^{0} = (0-0) - \left(-\tfrac{1}{3}(-1) - \tfrac{1}{2}(1)\right)$$
$$= -\tfrac{1}{3} + \tfrac{1}{2} = \tfrac{1}{6}$$

40.)



<u>Find L_1</u> :
$m = \frac{4-0}{6-4} = \frac{4}{2} = 2$ so
$Y - 4 = 2(X-6) \rightarrow$
$\boxed{Y = 2X - 8}$;

<u>Find L_2</u> : $m = \frac{4-0}{6-0} = \frac{2}{3}$ so $Y - 0 = \frac{2}{3}(X-0)$
$\rightarrow \boxed{Y = \frac{2}{3}X}$; so

$$\text{Area} = \int_0^4 \tfrac{2}{3}x\, dx + \int_4^6 \left(\tfrac{2}{3}x - (2x-8)\right) dx$$
$$= \tfrac{1}{3}x^2\Big|_0^4 + \int_4^6 \left(8 - \tfrac{4}{3}x\right) dx$$
$$= \tfrac{16}{3} + \left(8x - \tfrac{2}{3}x^2\right)\Big|_4^6$$
$$= \tfrac{16}{3} + \left(48 - \tfrac{72}{3}\right) - \left(32 - \tfrac{32}{3}\right)$$
$$= \tfrac{16}{3} + 16 - \tfrac{40}{3} = \tfrac{16}{3} + \tfrac{48}{3} - \tfrac{40}{3} = \tfrac{24}{3} = 8 \quad \text{OR}$$

$Y = 2X - 8 \rightarrow 2X = Y + 8 \rightarrow \boxed{X = \frac{1}{2}Y + 4}$; and

$Y = \frac{2}{3}X \rightarrow \boxed{X = \frac{3}{2}Y}$; then

$$\text{Area} = \int_0^4 \left[\left(\frac{1}{2}Y + 4\right) - \frac{3}{2}Y\right] dy$$

$$= \int_0^4 (4 - Y)\, dy = \left(4Y - \frac{1}{2}Y^2\right)\Big|_0^4 = (16 - 8) - (0 - 0)$$

$$= 8 .$$

Handout 7

1.) a.) $\int_0^{\frac{\pi}{6}} (1+\cos 3x)\,dx = \left(x + \frac{1}{3}\sin 3x\right)\Big|_0^{\frac{\pi}{6}}$

$= \frac{\pi}{6} + \frac{1}{3}\sin\frac{\pi}{2} = \frac{\pi}{6} + \frac{1}{3}$

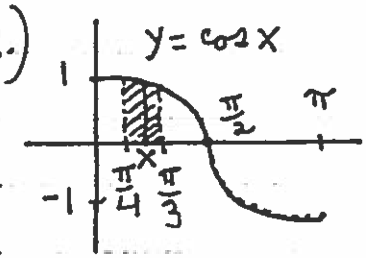
b.) $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} (\sec x \tan x - \sin x)\,dx$

$= (\sec x + \cos x)\Big|_{\frac{\pi}{4}}^{\frac{\pi}{3}}$

$= \left(\sec\frac{\pi}{3} + \cos\frac{\pi}{3}\right) - \left(\sec\frac{\pi}{4} + \cos\frac{\pi}{4}\right)$

$= \left(2 + \frac{1}{2}\right) - \left(\sqrt{2} + \frac{1}{\sqrt{2}}\right) = \frac{5}{2} - \sqrt{2} - \frac{1}{\sqrt{2}}$

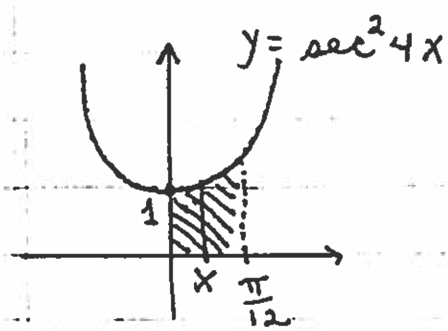
2.)



a.) $\text{Area} = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \cos x\,dx$

$= \sin x \Big|_{\frac{\pi}{4}}^{\frac{\pi}{3}}$

$= \sin\frac{\pi}{3} - \sin\frac{\pi}{4} = \boxed{\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}}$



b.) $\text{Area} = \int_0^{\frac{\pi}{12}} \sec^2 4x\,dx$

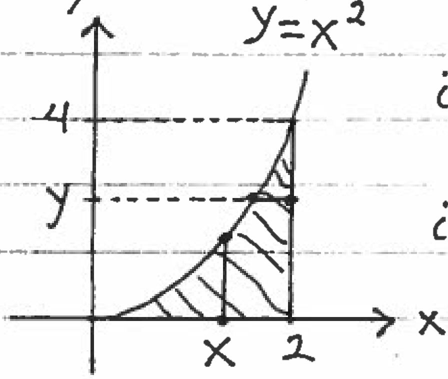
$= \frac{1}{4}\cdot\tan 4x \Big|_0^{\frac{\pi}{12}}$

$= \frac{1}{4}\tan\frac{\pi}{3} - \frac{1}{4}\cancel{\tan 0}^{\,0}$

$= \frac{1}{4}(\sqrt{3}) = \frac{\sqrt{3}}{4}$

3.) a.)

$x = \sqrt{y}$ or $y = x^2$

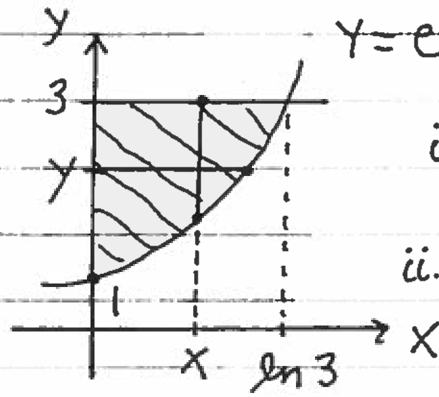


i.) $\text{Area} = \int_0^2 x^2\,dx$

ii.) $\text{Area} = \int_0^4 \left[2 - \sqrt{y}\right] dy$

b.)

$y = e^x$ or $x = \ln y$



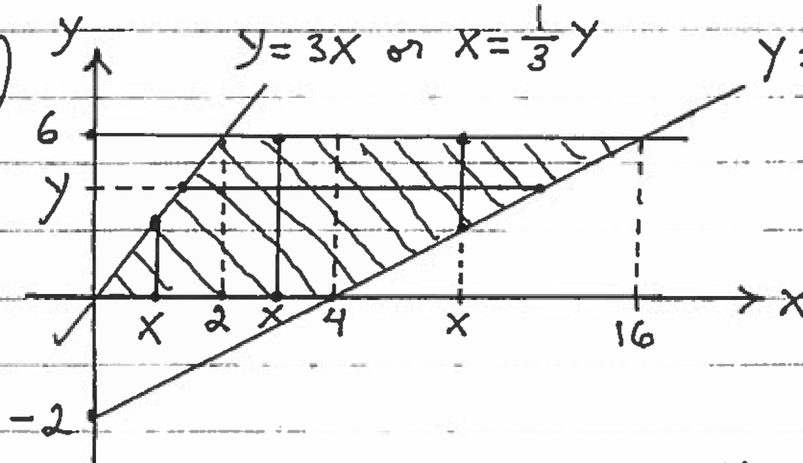
i.) $\text{Area} = \int_0^{\ln 3} \left[3 - e^x\right] dx$

ii.) $\text{Area} = \int_1^3 \ln y\,dy$

c.)

$y = 3x$ or $x = \frac{1}{3}y$

$y = \frac{1}{2}x - 2$ or $x = 2y + 4$

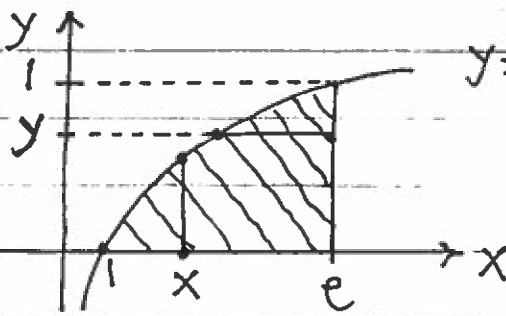


i.) $\text{Area} = \int_0^2 3x\,dx + \int_2^4 6\,dx + \int_4^{16} \left(6 - \left(\frac{1}{2}x - 2\right)\right) dx$

ii.) $\text{Area} = \int_0^6 \left((2y + 4) - \left(\frac{1}{3}y\right)\right) dy$

d.)

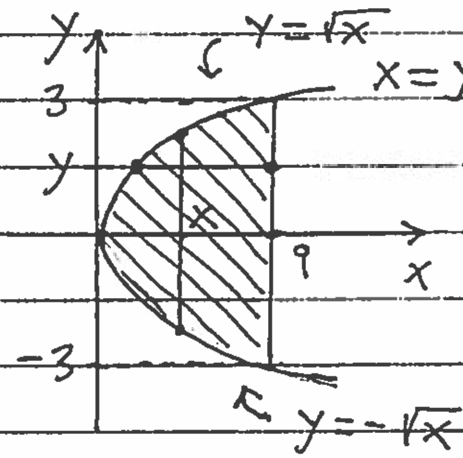
$y = \ln x$ or $x = e^y$



i.) $\text{Area} = \int_1^e \ln x \, dx$

ii.) $\text{Area} = \int_0^1 (e - e^y)\, dy$

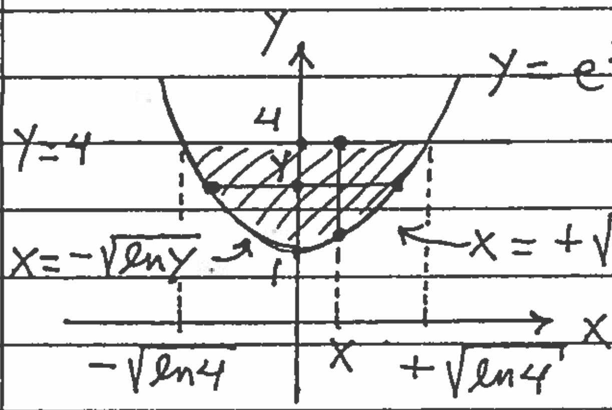
e.)



i.) $\text{Area} = \int_0^9 (\sqrt{x} - (-\sqrt{x}))\, dx$

ii.) $\text{Area} = \int_{-3}^{3} (9 - y^2)\, dy$

f.)



$y = e^{x^2} \rightarrow \ln y = x^2$

$\rightarrow x = \pm\sqrt{\ln y}$;

$4 = e^{x^2} \rightarrow$

$\ln 4 = x^2 \rightarrow$

$x = \pm\sqrt{\ln 4}$

i.) $\text{Area} = \int_{-\sqrt{\ln 4}}^{+\sqrt{\ln 4}} (4 - e^{x^2})\, dx$

ii.) $\text{Area} = \int_1^4 (\sqrt{\ln y} - (-\sqrt{\ln y}))\, dy$