

Math 16B

Kouba

Improper Integral

Example : $\int_3^{\infty} \frac{x+1}{x^2-2x} dx$

$$= \lim_{A \rightarrow \infty} \int_3^A \frac{x+1}{x(x-2)} dx$$

$$= \lim_{A \rightarrow \infty} \int_3^A \left[\frac{B}{x} + \frac{C}{x-2} \right] dx$$

$$(x+1 = B(x-2) + Cx)$$

Let $x=0$: $1 = B(-2) + C(0) \rightarrow B = -1/2$

Let $x=2$: $3 = B(0) + C(2) \rightarrow C = 3/2$

$$= \lim_{A \rightarrow \infty} \int_3^A \left[\frac{-1/2}{x} + \frac{3/2}{x-2} \right] dx$$

$$= \lim_{A \rightarrow \infty} \left(-\frac{1}{2} \ln|x| + \frac{3}{2} \ln|x-2| \right) \Big|_3^A$$

$$= \lim_{A \rightarrow \infty} \left[\left(-\frac{1}{2} \ln|A| + \frac{3}{2} \ln|A-2| \right) - \left(-\frac{1}{2} \ln 3 + \frac{3}{2} \ln 1 \right) \right]$$

\downarrow
0

$$= \lim_{A \rightarrow \infty} \left[-\ln |A|^{\frac{1}{2}} + \ln |A-2|^{\frac{3}{2}} + \frac{1}{2} \ln 3 \right]$$

$$= \lim_{A \rightarrow \infty} \ln \left(\frac{|A-2|^{\frac{3}{2}}}{|A|^{\frac{1}{2}}} \right) + \frac{1}{2} \ln 3$$

$$= \lim_{A \rightarrow \infty} \ln \left(\frac{|A-2|^{\frac{3}{2}}}{|A^{\frac{1}{3}}|^{\frac{3}{2}}} \right) + \frac{1}{2} \ln 3$$

$$= \lim_{A \rightarrow \infty} \ln \left| \frac{A-2}{A^{\frac{1}{3}}} \right|^{\frac{3}{2}} + \frac{1}{2} \ln 3$$

$$= \lim_{A \rightarrow \infty} \ln \left| A^{\frac{2}{3}} - \frac{2}{A^{\frac{1}{3}}} \right|^{\frac{3}{2}} + \frac{1}{2} \ln 3$$

$$= \ln (\infty - 0)^{\frac{3}{2}} + \frac{1}{2} \ln 3$$

$$= \infty + \frac{1}{2} \ln 3$$

$$= \infty$$