

In the table of integrals below and on the next two pages, the formulas have been grouped into eight different types according to the form of the integrand.

Forms involving u^n

Forms involving $a + bu$

Forms involving $\sqrt{a + bu}$

Forms involving $\sqrt{u^2 \pm a^2}$

Forms involving $u^2 - a^2$

Forms involving $\sqrt{a^2 - u^2}$

Forms involving e^u

Forms involving $\ln u$

Table of Integrals

Forms involving u^n

$$1. \int u^n du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$$

$$2. \int \frac{1}{u} du = \ln|u| + C$$

Forms involving $a + bu$

$$3. \int \frac{u}{a + bu} du = \frac{1}{b^2}(bu - a \ln|a + bu|) + C$$

$$4. \int \frac{u}{(a + bu)^2} du = \frac{1}{b^2} \left(\frac{a}{a + bu} + \ln|a + bu| \right) + C$$

$$5. \int \frac{u}{(a + bu)^n} du = \frac{1}{b^2} \left[\frac{-1}{(n-2)(a + bu)^{n-2}} + \frac{a}{(n-1)(a + bu)^{n-1}} \right] + C, \quad n \neq 1, 2$$

$$6. \int \frac{u^2}{a + bu} du = \frac{1}{b^3} \left[-\frac{bu}{2}(2a - bu) + a^2 \ln|a + bu| \right] + C$$

$$7. \int \frac{u^2}{(a + bu)^2} du = \frac{1}{b^3} \left(bu - \frac{a^2}{a + bu} - 2a \ln|a + bu| \right) + C$$

$$8. \int \frac{u^2}{(a + bu)^3} du = \frac{1}{b^3} \left[\frac{2a}{a + bu} - \frac{a^2}{2(a + bu)^2} + \ln|a + bu| \right] + C$$

$$9. \int \frac{u^2}{(a + bu)^n} du = \frac{1}{b^3} \left[\frac{-1}{(n-3)(a + bu)^{n-3}} + \frac{2a}{(n-2)(a + bu)^{n-2}} - \frac{a^2}{(n-1)(a + bu)^{n-1}} \right] + C, \quad n \neq 1, 2, 3$$

$$10. \int \frac{1}{u(a + bu)} du = \frac{1}{a} \ln \left| \frac{u}{a + bu} \right| + C$$

$$11. \int \frac{1}{u(a + bu)^2} du = \frac{1}{a} \left(\frac{1}{a + bu} + \frac{1}{a} \ln \left| \frac{u}{a + bu} \right| \right) + C$$

$$12. \int \frac{1}{u^2(a + bu)} du = -\frac{1}{a} \left(\frac{1}{u} + \frac{b}{a} \ln \left| \frac{u}{a + bu} \right| \right) + C$$

$$13. \int \frac{1}{u^2(a + bu)^2} du = -\frac{1}{a^2} \left[\frac{a + 2bu}{u(a + bu)} + \frac{2b}{a} \ln \left| \frac{u}{a + bu} \right| \right] + C$$

Table of Integrals (continued)

Forms involving $\sqrt{a+bu}$

$$14. \int u^n \sqrt{a+bu} \, du = \frac{2}{b(2n+3)} \left[u^n (a+bu)^{3/2} - na \int u^{n-1} \sqrt{a+bu} \, du \right]$$

$$15. \int \frac{1}{u\sqrt{a+bu}} \, du = \frac{1}{\sqrt{a}} \ln \left| \frac{\sqrt{a+bu} - \sqrt{a}}{\sqrt{a+bu} + \sqrt{a}} \right| + C, \quad a > 0$$

$$16. \int \frac{1}{u^n \sqrt{a+bu}} \, du = \frac{-1}{a(n-1)} \left[\frac{\sqrt{a+bu}}{u^{n-1}} + \frac{(2n-3)b}{2} \int \frac{1}{u^{n-1} \sqrt{a+bu}} \, du \right], \quad n \neq 1$$

$$17. \int \frac{\sqrt{a+bu}}{u} \, du = 2\sqrt{a+bu} + a \int \frac{1}{u\sqrt{a+bu}} \, du$$

$$18. \int \frac{\sqrt{a+bu}}{u^n} \, du = \frac{-1}{a(n-1)} \left[\frac{(a+bu)^{3/2}}{u^{n-1}} + \frac{(2n-5)b}{2} \int \frac{\sqrt{a+bu}}{u^{n-1}} \, du \right], \quad n \neq 1$$

$$19. \int \frac{u}{\sqrt{a+bu}} \, du = \frac{2(2a-bu)}{3b^2} \sqrt{a+bu} + C$$

$$20. \int \frac{u^n}{\sqrt{a+bu}} \, du = \frac{2}{(2n+1)b} \left(u^n \sqrt{a+bu} - na \int \frac{u^{n-1}}{\sqrt{a+bu}} \, du \right)$$

Forms involving $\sqrt{u^2 \pm a^2}$, $a > 0$

$$21. \int \sqrt{u^2 \pm a^2} \, du = \frac{1}{2} (u\sqrt{u^2 \pm a^2} \pm a^2 \ln|u + \sqrt{u^2 \pm a^2}|) + C$$

$$22. \int u^2 \sqrt{u^2 \pm a^2} \, du = \frac{1}{8} [u(2u^2 \pm a^2) \sqrt{u^2 \pm a^2} - a^4 \ln|u + \sqrt{u^2 \pm a^2}|] + C$$

$$23. \int \frac{\sqrt{u^2 + a^2}}{u} \, du = \sqrt{u^2 + a^2} - a \ln \left| \frac{a + \sqrt{u^2 + a^2}}{u} \right| + C$$

$$24. \int \frac{\sqrt{u^2 \pm a^2}}{u^2} \, du = \frac{-\sqrt{u^2 \pm a^2}}{u} + \ln|u + \sqrt{u^2 \pm a^2}| + C$$

$$25. \int \frac{1}{\sqrt{u^2 \pm a^2}} \, du = \ln|u + \sqrt{u^2 \pm a^2}| + C$$

$$26. \int \frac{1}{u\sqrt{u^2 + a^2}} \, du = -\frac{1}{a} \ln \left| \frac{a + \sqrt{u^2 + a^2}}{u} \right| + C$$

$$27. \int \frac{u^2}{\sqrt{u^2 \pm a^2}} \, du = \frac{1}{2} (u\sqrt{u^2 \pm a^2} \mp a^2 \ln|u + \sqrt{u^2 \pm a^2}|) + C$$

$$28. \int \frac{1}{u^2 \sqrt{u^2 \pm a^2}} \, du = \mp \frac{\sqrt{u^2 \pm a^2}}{a^2 u} + C$$

Table of Integrals (continued)*Forms involving $u^2 - a^2$, $a > 0$*

$$29. \int \frac{1}{u^2 - a^2} du = - \int \frac{1}{a^2 - u^2} du = \frac{1}{2a} \ln \left| \frac{u - a}{u + a} \right| + C$$

$$30. \int \frac{1}{(u^2 - a^2)^n} du = \frac{-1}{2a^2(n-1)} \left[\frac{u}{(u^2 - a^2)^{n-1}} + (2n-3) \int \frac{1}{(u^2 - a^2)^{n-1}} du \right], \quad n \neq 1$$

Forms involving $\sqrt{a^2 - u^2}$, $a > 0$

$$31. \int \frac{\sqrt{a^2 - u^2}}{u} du = \sqrt{a^2 - u^2} - a \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right| + C$$

$$32. \int \frac{1}{u\sqrt{a^2 - u^2}} du = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right| + C$$

$$33. \int \frac{1}{u^2\sqrt{a^2 - u^2}} du = \frac{-\sqrt{a^2 - u^2}}{a^2 u} + C$$

Forms involving e^u

$$34. \int e^u du = e^u + C$$

$$35. \int u e^u du = (u - 1)e^u + C$$

$$36. \int u^n e^u du = u^n e^u - n \int u^{n-1} e^u du$$

$$37. \int \frac{1}{1 + e^u} du = u - \ln(1 + e^u) + C$$

$$38. \int \frac{1}{1 + e^{nu}} du = u - \frac{1}{n} \ln(1 + e^{nu}) + C$$

Forms involving $\ln u$

$$39. \int \ln u du = u(-1 + \ln u) + C$$

$$40. \int u \ln u du = \frac{u^2}{4}(-1 + 2 \ln u) + C$$

$$41. \int u^n \ln u du = \frac{u^{n+1}}{(n+1)^2}[-1 + (n+1) \ln u] + C, \quad n \neq -1$$

$$42. \int (\ln u)^2 du = u[2 - 2 \ln u + (\ln u)^2] + C$$

$$43. \int (\ln u)^n du = u(\ln u)^n - n \int (\ln u)^{n-1} du$$