Example: What should n be so that M_n, the Midpoint Rule with n rectangles, estimates the exact value of $\int_{-3}^{2} e^{-x^2}\,dx$ with an absolute Error at most 0.01?

Solution: $f(x) = e^{-x^2} \xrightarrow{D}$

$f'(x) = -2xe^{-x^2} = \dfrac{-2x}{e^{x^2}}$; then

$$\max_{-3 \le x \le 2} |f'(x)| = \max_{-3 \le x \le 2} \left| \frac{-2x}{e^{x^2}} \right|$$

$$= \max_{-3 \le x \le 2} \frac{2|x|}{e^{x^2}}$$

$$\le \frac{2|-3|}{e^{(0)^2}} = 6 \text{ ; and}$$

$h = \frac{b-a}{n} = \frac{2-(-3)}{n} = \frac{5}{n}$; the absolute Midpoint Error is

$$|E_n| \le (b-a)h \left\{ \max_{a \le x \le b} |f'(x)| \right\}$$

$$= (2-(-3)) \cdot \left(\frac{5}{n}\right) \{6\}$$

$$= \frac{150}{n}, \quad \text{i.e.,} \quad |E_n| \le \frac{150}{n} ;$$

now use the given error:

$$|E_n| \le \frac{150}{n} \le 0.01 \longrightarrow$$

$$\frac{150}{1/100} \le n \longrightarrow$$

$n \ge 15{,}000$ works.