**Musical Pitch**

The pitch of a musical note is determined by the frequency of the vibration which causes it. Middle C on the piano, for example, corresponds to a vibration of 263 hertz (cycles per second). A note one octave above middle C vibrates at 526 hertz, and a note two octaves above middle C vibrates at 1052 hertz. (See Table 1.7.)

<table>
<thead>
<tr>
<th>Number, $n$, of octaves above middle C</th>
<th>Number of hertz $V = f(n)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>263</td>
</tr>
<tr>
<td>1</td>
<td>526</td>
</tr>
<tr>
<td>2</td>
<td>1052</td>
</tr>
<tr>
<td>3</td>
<td>2104</td>
</tr>
<tr>
<td>4</td>
<td>4208</td>
</tr>
</tbody>
</table>

**Table 1.7 Pitch of notes above middle C**

<table>
<thead>
<tr>
<th>$n$</th>
<th>$V = 263 \cdot 2^n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-3$</td>
<td>$263 \cdot 2^{-3} = 263(1/2^3) = 32.875$</td>
</tr>
<tr>
<td>$-2$</td>
<td>$263 \cdot 2^{-2} = 263(1/2^2) = 65.75$</td>
</tr>
<tr>
<td>$-1$</td>
<td>$263 \cdot 2^{-1} = 263(1/2) = 131.5$</td>
</tr>
<tr>
<td>0</td>
<td>$263 \cdot 2^0 = 263$</td>
</tr>
</tbody>
</table>

**Table 1.8 Pitch of notes below middle C**

Notice that

\[
\frac{526}{263} = 2 \quad \text{and} \quad \frac{1052}{526} = 2 \quad \text{and} \quad \frac{2104}{1052} = 2
\]

and so on. In other words, each value of $V$ is twice the value before, so

\[
f(1) = 526 = 263 \cdot 2 = 263 \cdot 2^1
\]
\[
f(2) = 1052 = 2 \cdot 263 = 2 \cdot 263 \cdot 2 = 263 \cdot 2^2
\]
\[
f(3) = 2104 = 2 \cdot 1052 = 2 \cdot 2 \cdot 263 = 2 \cdot 263 \cdot 2 = 263 \cdot 2^3.
\]

In general

\[
V = f(n) = 263 \cdot 2^n.
\]

The base 2 represents the fact that as we go up an octave, the frequency of vibrations doubles. Indeed, our ears hear a note as one octave higher than another precisely because it vibrates twice as fast. For the negative values of $n$ in Table 1.8, this function represents the octaves below middle C. The notes on a piano are represented by values of $n$ between $-3$ and $4$, and the human ear finds values of $n$ between $-4$ and 7 audible.

Although $V = f(n) = 263 \cdot 2^n$ makes sense in musical terms only for certain values of $n$, values of the function $f(x) = 263 \cdot 2^x$ can be calculated for all real $x$, and its graph has the typical exponential shape, as can be seen in Figure 1.19. It is concave up, climbing faster and faster as $x$ increases.

![Figure 1.19: Pitch as a function of number of octaves above middle C](image-url)