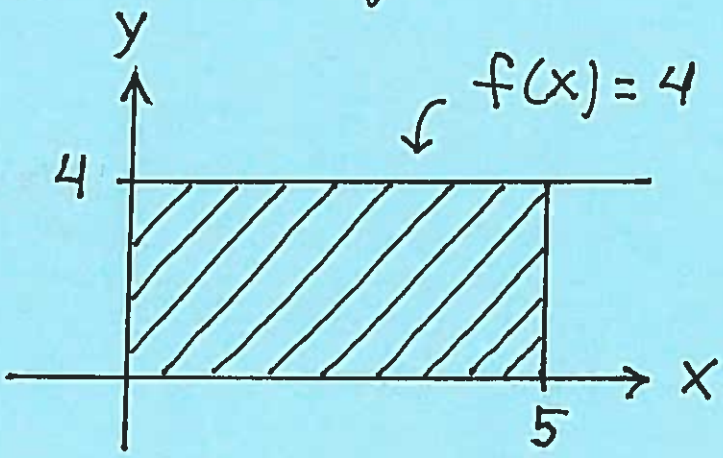
Areas of Geometric Shapes and Definite Integrals

Example:

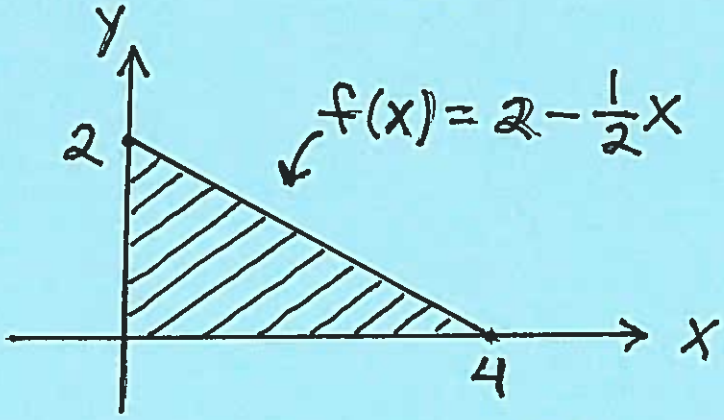


Area of $\square$ $= (5)(4) = 20$ and

$$\int_0^5 f(x)\,dx = \int_0^5 4\,dx = 4x\Big|_0^5$$

$$= 4(5) - 4(0) = 20$$

Example:



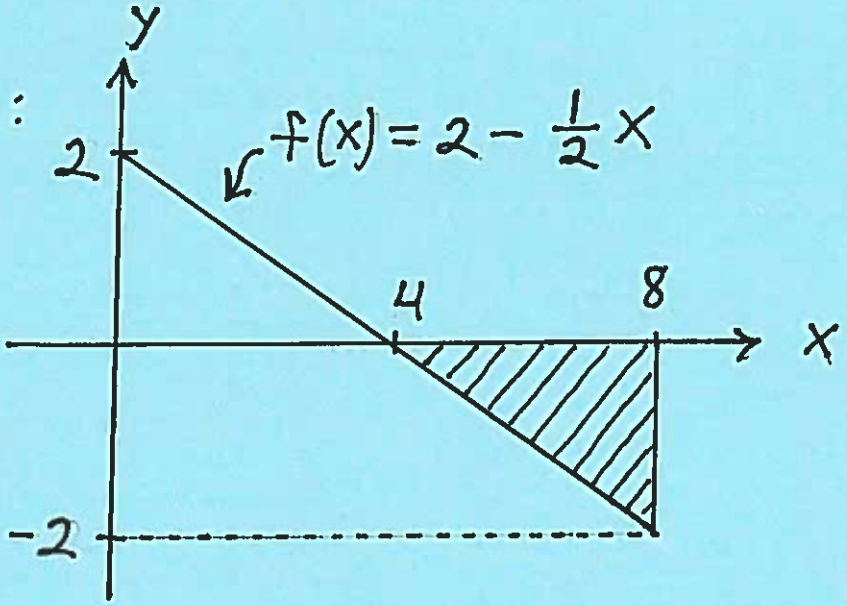
Area of $\triangle$ $= \frac{1}{2}(4)(2) = 4$ and

$$\int_0^4 f(x)\,dx = \int_0^4 \left(2 - \tfrac{1}{2}x\right) dx$$

$$= \left(2x - \tfrac{1}{4}x^2\right)\Big|_0^4 = \left(2(4) - \tfrac{1}{4}(4)^2\right) - \left(2(0) - \tfrac{1}{4}(0)^2\right)$$

$$= 8 - 4 - 0 = 4 \quad .$$

Example:



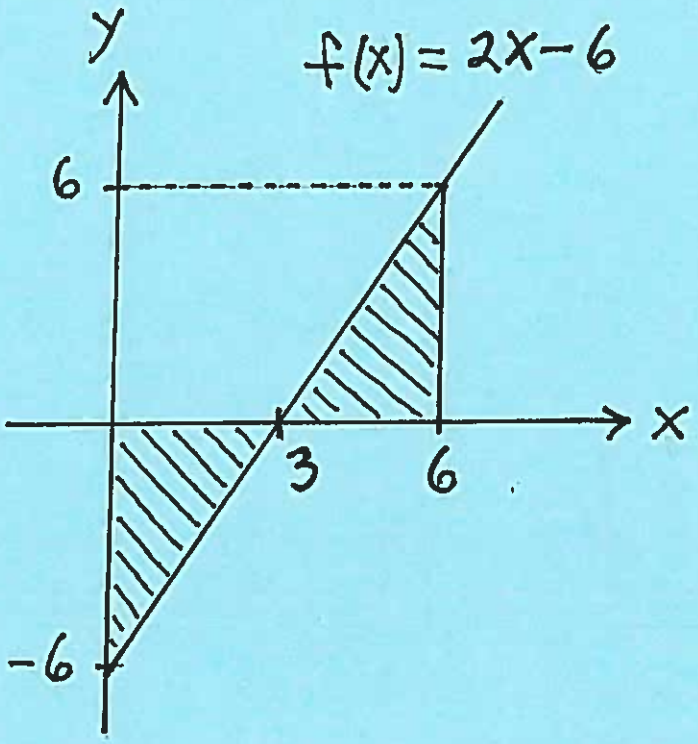
Area of ◺ $= \frac{1}{2}(4)(2) = 4$, and

$$\int_4^8 \left(2 - \tfrac{1}{2}x\right) dx = \left(2x - \tfrac{1}{4}x^2\right)\Big|_4^8$$

$$= \left(2(8) - \tfrac{1}{4}(8)^2\right) - \left(2(4) - \tfrac{1}{4}(4)^2\right)$$

$$= 16 - 16 - (8 - 4) = -4 \; !$$

Example:



Area of $\nabla + \Delta = \frac{1}{2}(3)(6) + \frac{1}{2}(3)(6) = 18$

and $\int_0^6 f(x)\,dx = \int_0^6 (2x-6)\,dx$

$= (x^2 - 6x)\Big|_0^6 = ((6)^2 - 6(6)) - ((0)^2 - 6(0))$

$= 36 - 36 - 0 = 0$!

CONCLUSION :

I.) If $f(x) \geq 0$, then

$$\int_a^b f(x)\,dx = \text{AREA}$$

II.) If $f(x) \leq 0$, then

$$\int_a^b f(x)\,dx = -\text{AREA}$$