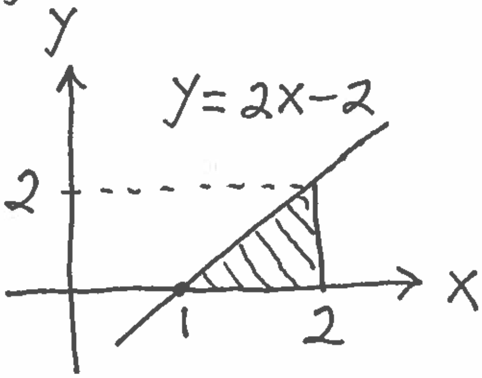
Example: Show that each function is a p.d.f.

1.) $f(x) = 2x - 2$ on $[1, 2]$:



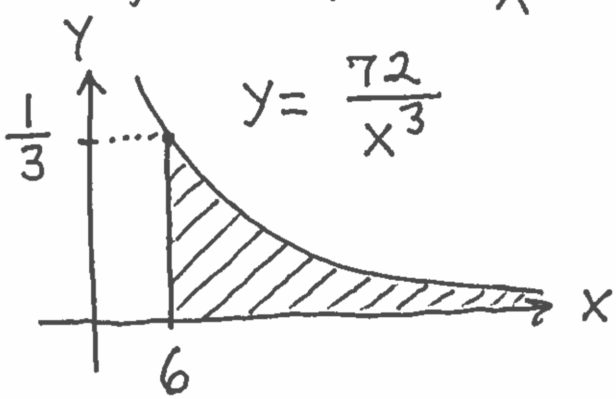
i.) Clearly $f(x) \geq 0$ on $[1, 2]$

ii.) $\int_1^2 (2x-2)\,dx$

$= (x^2 - 2x)\Big|_1^2$

$= (4-4) - (1-2) = +1$

2.) $f(x) = \frac{72}{x^3}$ for $x \geq 6$:



i.) Clearly, $f(x) \geq 0$ for $x \geq 6$

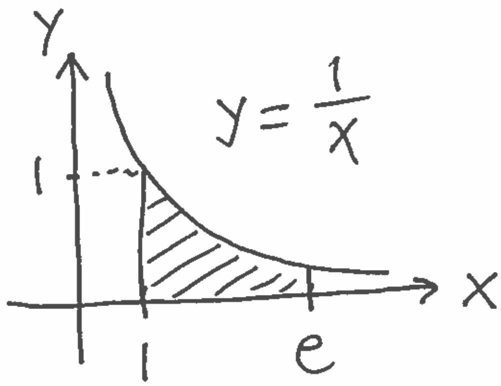
ii.) $\int_6^\infty \frac{72}{x^3}\,dx = \lim_{A \to \infty} \int_6^A 72x^{-3}\,dx$

$$= \lim_{A\to\infty} 72 \cdot \frac{1}{-2} x^{-2} \Big|_6^A$$

$$= \lim_{A\to\infty} \left(\frac{-36}{A^2} - \frac{-36}{6^2} \right)$$

$$= \frac{-36}{\infty} + 1 = 0 + 1 = 1$$

3.) $f(x) = \frac{1}{x}$ on $[1, e]$:

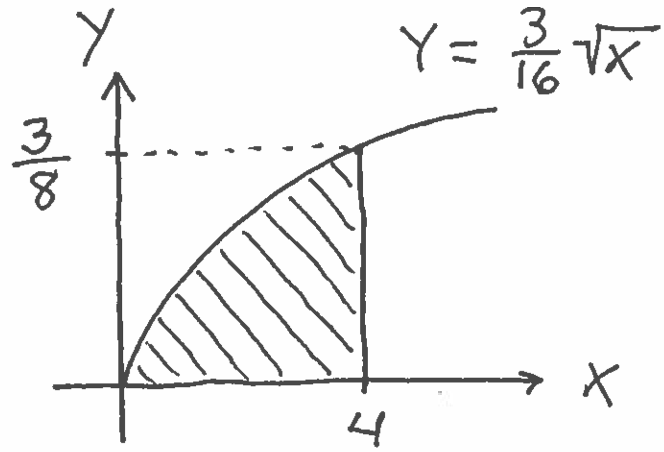


i.) Clearly, $f(x) \geq 0$ on $[1, e]$

ii.) $\int_1^e \frac{1}{x}\,dx = \ln|x| \Big|_1^e$

$$= \ln e - \ln 1 = 1 - 0 = 1$$

4.) $f(x) = \frac{3}{16}\sqrt{x}$ on $[0, 4]$:



i.) Clearly,

$f(x) \geq 0$ on [0,4]

ii.) $\int_0^4 \frac{3}{16}\sqrt{x}\,dx = \frac{3}{16}\cdot\frac{2}{3}x^{3/2}\Big|_0^4$

$= \frac{1}{8}(4)^{3/2} - \frac{1}{8}(0)^{3/2}$

$= \frac{1}{8}(8) - 0$

$= 1$