Math 16B
Section 5.1

Antiderivatives

They can be used to compute area, length, volume, probability, force, kinetic energy, etc.

Definition: If $F'(x) = f(x)$, i.e., the derivative of $F(x)$ is $f(x)$, then we say that an antiderivative for $f(x)$ is $F(x)$.

Example: 1.) $D(x^2+x) = 2x+1$, so x^2+x is an antiderivative for $2x+1$.

2.) $D(\sin x) = \cos x$, so $\sin x$ is an antiderivative for $\cos x$.

3.) $D(5+\sin x) = \cos x$, so $5+\sin x$ is also an antiderivative for $\cos x$.

FACT: If $F(x)$ is an antiderivative for $f(x)$, then so is $F(x)+c$, since

$$D\big(F(x)\big) = f(x)$$

and

$$D\big(F(x)+c\big) = f(x)+0 = f(x).$$

NOTATION: $\int f(x)\,dx = F(x)+c$ means $F(x)+c$ is the most general antiderivative for $f(x)$.

Example: $\int 2x\,dx = x^2+c$ since $D(x^2+c) = 2x$.

Example: Determine the following antiderivatives (also called indefinite integrals).

1.) $\int 3\,dx = 3x+c$

2.) $\int -\frac{1}{2}\,dx = -\frac{1}{2}x+c$

RULE: $\int k\,dx = kx+c$

3.) $\int x \, dx = \frac{1}{2}x^2 + c$

4.) $\int x^2 dx = \frac{1}{3}x^3 + c$

5.) $\int x^{1/2} = \frac{2}{3}x^{3/2} + c$

6.) $\int x^{-2} dx = -x^{-1} + c$

7.) $\int x^{-5} dx = \frac{-1}{4}x^{-4} + c$

<u>RULE</u>: $\int x^n dx = \frac{1}{n+1}x^{n+1} + c, \; n \neq -1$

8.) $\int (7-x) dx = 7x - \frac{1}{2}x^2 + c$

9.) $\int x^5(3+x) dx = \int (3x^5 + x^6) dx$

$= 3 \cdot \frac{1}{6}x^6 + \frac{1}{7}x^7 + c$

10.) $\int \frac{x^4 + 2x^3 - 1}{x^3} dx$

$= \int \left[\frac{x^4}{x^3} + \frac{2x^3}{x^3} - \frac{1}{x^3}\right] dx$

$= \int [x + 2 - x^{-3}] dx = \frac{1}{2}x^2 + 2x - \frac{-1}{2}x^{-2} + c$

11.) $\int \frac{(2-x)(x+4)}{\sqrt{x}}\,dx = \int \frac{2x+8-x^2-4x}{x^{1/2}}\,dx$

$= \int \frac{8-2x-x^2}{x^{1/2}}\,dx = \int \left[\frac{8}{x^{1/2}} - \frac{2x}{x^{1/2}} - \frac{x^2}{x^{1/2}}\right] dx$

$= \int \left[8x^{-1/2} - 2x^{1/2} - x^{3/2}\right] dx = 8 \cdot \frac{1}{1/2} x^{1/2} - 2 \cdot \frac{1}{3/2} x^{3/2} - \frac{1}{5/2} x^{5/2} + C$

RECALL: (from Math 16A)

If $s(t)$ is distance at time t, then $s'(t) = v(t)$ is velocity at time t, and $s''(t) = a(t)$ is acceleration at time t.

Derive all needed formulas for the following problem. Begin by assuming that the acceleration due to gravity is

$$s''(t) = -32 \text{ ft./sec.}^2$$

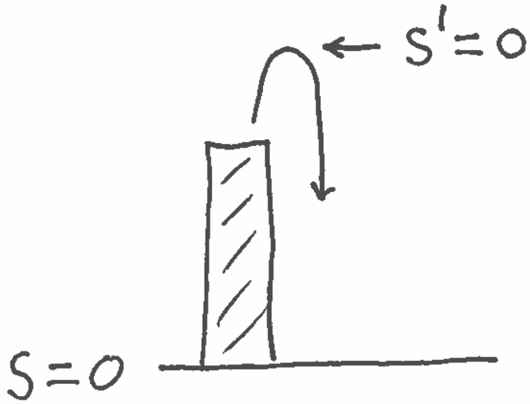
Example: a ball is thrown upward from the top of a 128-ft. high building with initial velocity 32 ft./sec.

1.) How high does the ball go?

2.) How long is the ball in the air?

3.) What is the ball's velocity as it strikes the ground.

Assume that $s(t)$ is the height of the ball above the ground at time t.



$$s''(t) = -32 \xrightarrow{(A.D.)}$$

$$s'(t) = -32t + c$$

(and $t=0$, $s' = 32 \rightarrow$

$32 = -32(0) + c \rightarrow c = 32$)

$$\rightarrow \boxed{s'(t) = -32t + 32} \xrightarrow{(A.D.)}$$

$$s(t) = -32 \cdot \tfrac{1}{2} t^2 + 32t + c$$

(and $t=0$, $s = 128 \rightarrow$

$128 = -16(0)^2 + 32(0) + c \rightarrow c = 128$)

$$\rightarrow \boxed{s(t) = -16t^2 + 32t + 128}$$

1.) highest point: $s'(t) = 0 \longrightarrow$

$-32t + 32 = 0 \longrightarrow \boxed{t = 1} \longrightarrow$

height $s(1) = -16(1)^2 + 32(1) + 128$

$= \boxed{144 \text{ ft.}}$

2.) strike ground: $s(t) = 0 \longrightarrow$

$-16t^2 + 32t + 128 = 0 \longrightarrow$

$-16(t^2 - 2t - 8) = 0 \longrightarrow$

$-16(t-4)(t+2) = 0 \longrightarrow$

$\boxed{t = 4}$ or $t = -2$ (NO!)

seconds

3.) velocity striking ground:

$s'(4) = -32(4) + 32 = -96$ ft./sec.