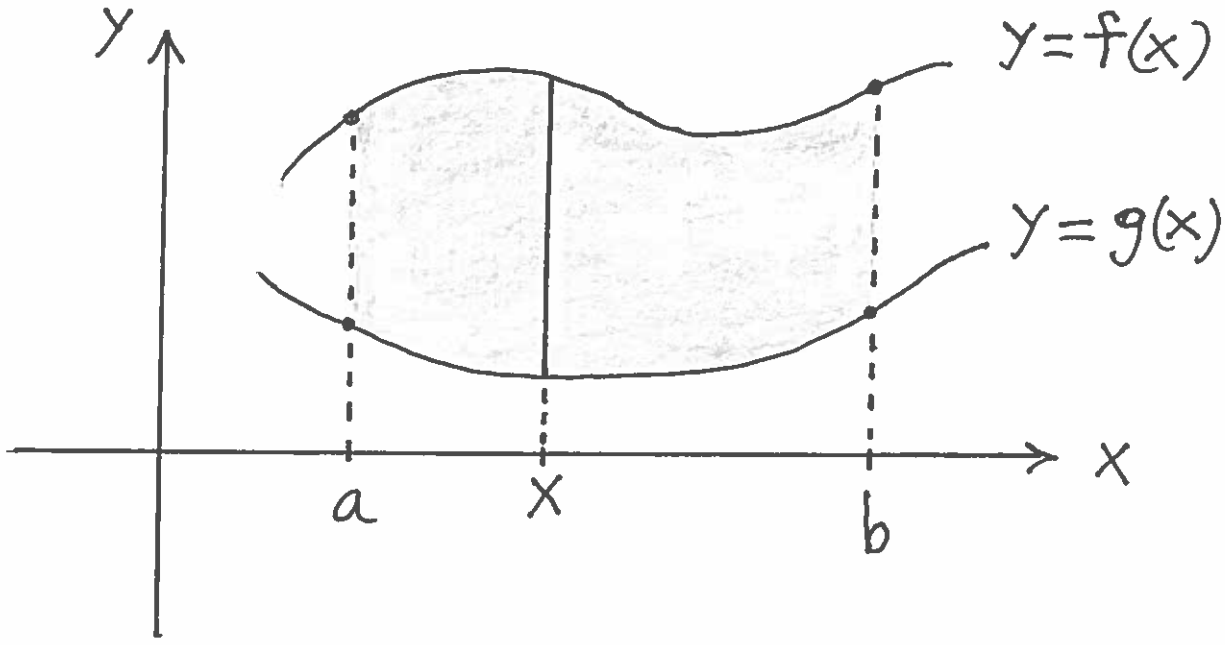
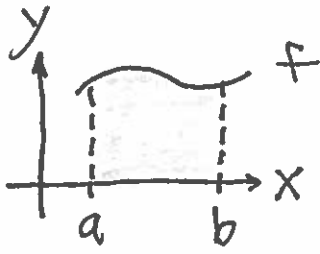
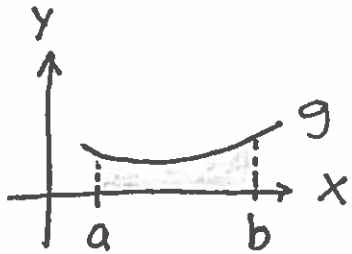


Math 16B
Section 5.5

Area Between Curves

Assume that $f(x) \geq g(x)$ for $a \leq x \leq b$.
Find the AREA of the shaded region below:



AREA =  - 

The equation shows the area between the curves as the difference between the area under the upper curve $f(x)$ and the area under the lower curve $g(x)$ from $x=a$ to $x=b$. Each diagram shows a shaded region bounded by the curve, the x-axis, and vertical dashed lines at $x=a$ and $x=b$.

(SEE next page.)

$$= \int_a^b f(x) dx - \int_a^b g(x) dx$$

$$= \int_a^b (f(x) - g(x)) dx, \text{ i.e.,}$$

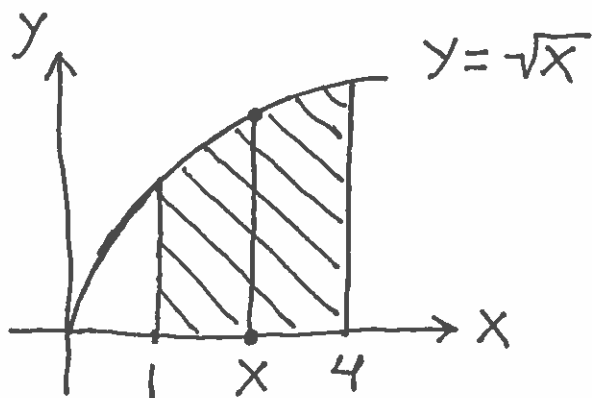
$$\text{AREA} = \int_a^b (f(x) - g(x)) dx$$

↑ ↑
TOP BOTTOM

Example: Find the AREA of the region bounded by the graphs of the given equations (SET UP ONLY)

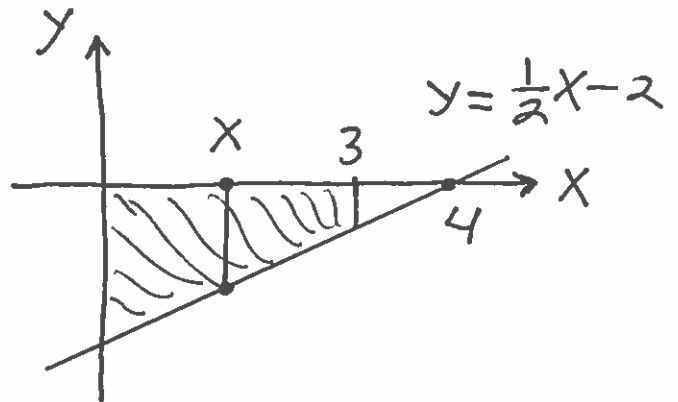
1.) $y = \sqrt{x}$, $y = 0$, $x = 1$, $x = 4$

$$\begin{aligned} \text{AREA} &= \int_1^4 (\sqrt{x} - 0) dx \\ &= \int_1^4 \sqrt{x} dx \end{aligned}$$



$$2.) y = \frac{1}{2}x - 2, y = 0, x = 0, x = 3$$

$$\begin{aligned} \text{AREA} &= \int_0^3 (0 - (\frac{1}{2}x - 2)) dx \\ &= \int_0^3 (2 - \frac{1}{2}x) dx \end{aligned}$$



$$3.) y = x^2, y = x + 6$$

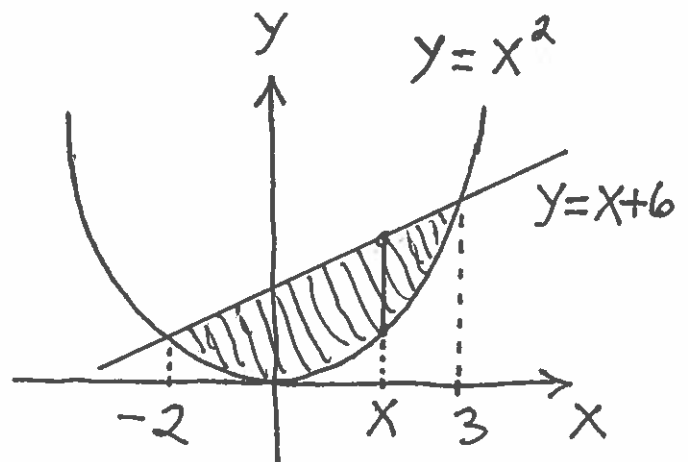
$$x^2 = x + 6 \rightarrow$$

$$x^2 - x - 6 = 0 \rightarrow$$

$$(x - 3)(x + 2) = 0$$

$$\begin{array}{ccc} \downarrow & & \downarrow \\ x = 3 & & x = -2 \end{array}$$

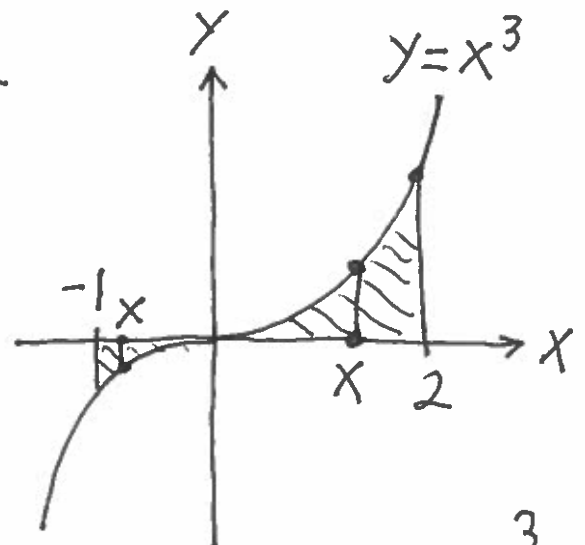
$$\text{AREA} = \int_{-2}^3 [x + 6 - x^2] dx$$



$$4.) y = x^3, y = 0, x = -1, x = 2$$

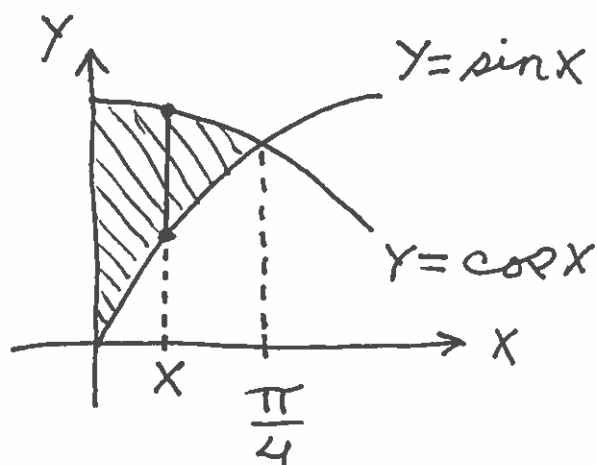
$$\text{AREA} = \int_{-1}^0 (0 - x^3) dx$$

$$+ \int_0^2 (x^3 - 0) dx$$



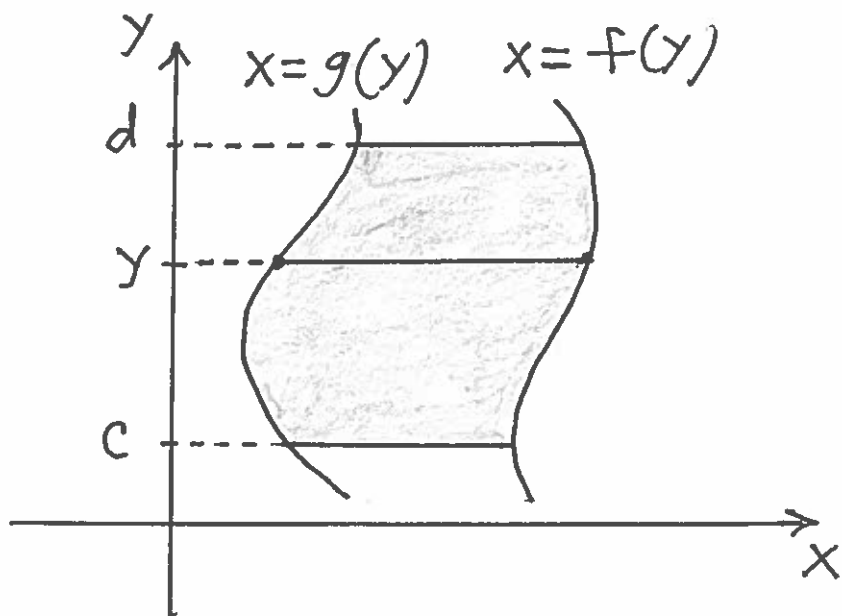
5.) $y = \sin x, y = \cos x, x = 0$

$$\text{AREA} = \int_0^{\frac{\pi}{4}} (\cos x - \sin x) dx$$



Variation of Area Between Curves

Assume that $f(y) \geq g(y)$ for $c \leq y \leq d$.



Area of shaded region is

$$\text{AREA} = \int_c^d f(y) dy - \int_c^d g(y) dy$$

(SEE next page.)

$$= \int_c^d (f(y) - g(y)) dy, \text{ i.e.,}$$

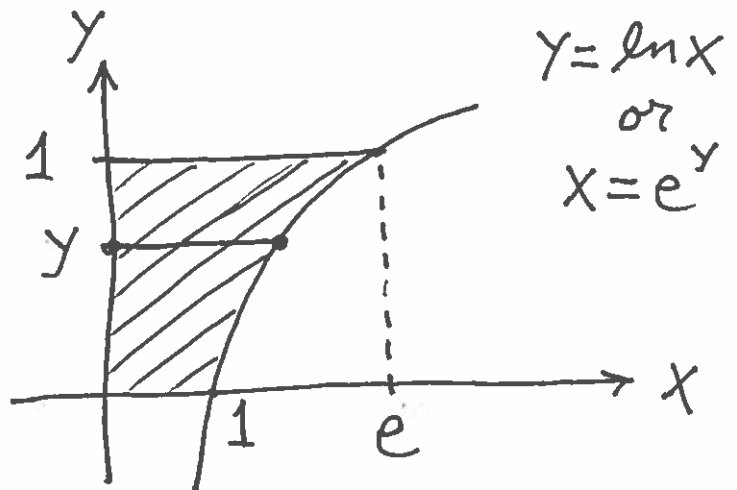
$$\text{AREA} = \int_c^d (f(y) - g(y)) dy$$

↑
↑

RIGHT
LEFT

6.) $y = \ln x, y = 0, y = 1, x = 0$

$$\text{AREA} = \int_0^1 (e^y - 0) dy$$



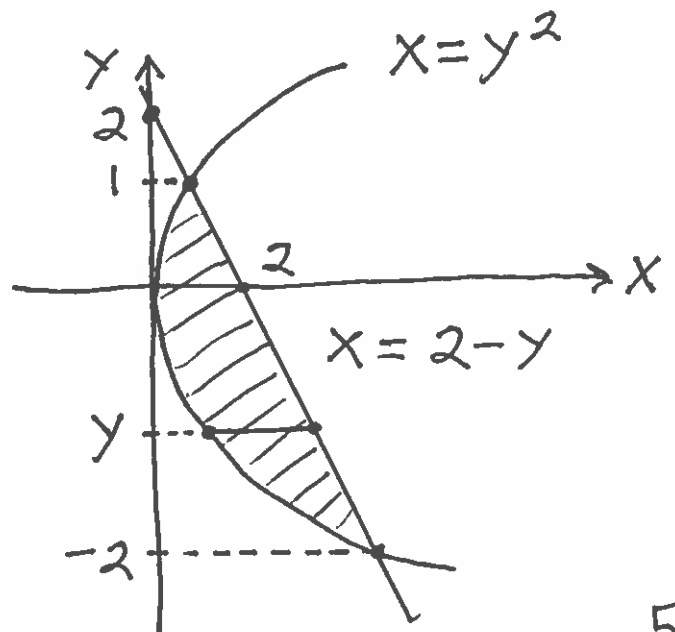
7.) $x = 2 - y, x = y^2$

$$y^2 = 2 - y \rightarrow$$

$$y^2 + y - 2 = 0 \rightarrow$$

$$(y-1)(y+2) = 0 \rightarrow y = 1, \\ y = -2$$

$$\text{AREA} = \int_{-2}^1 (2 - y - y^2) dy$$



Example: Find the AREA of the region bounded by the graphs of the given equations (SET UP ONLY) using

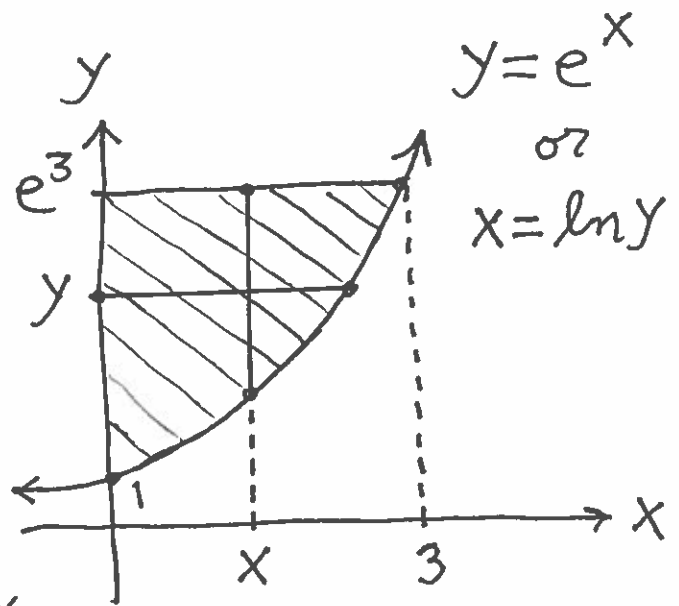
a.) VERTICAL cross-sections.

b.) HORIZONTAL cross-sections.

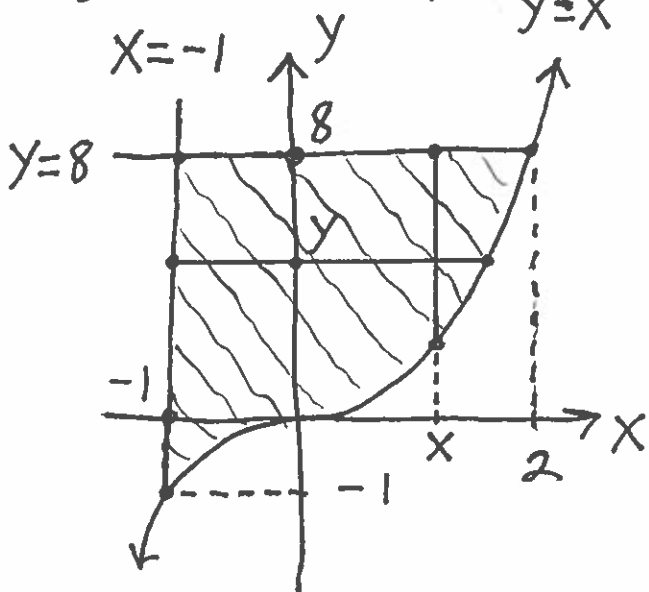
1.) $y = e^x, y = e^3, x = 0$

a.) $AREA = \int_0^3 (e^3 - e^x) dx$

b.) $AREA = \int_1^{e^3} (\ln y - 0) dy$



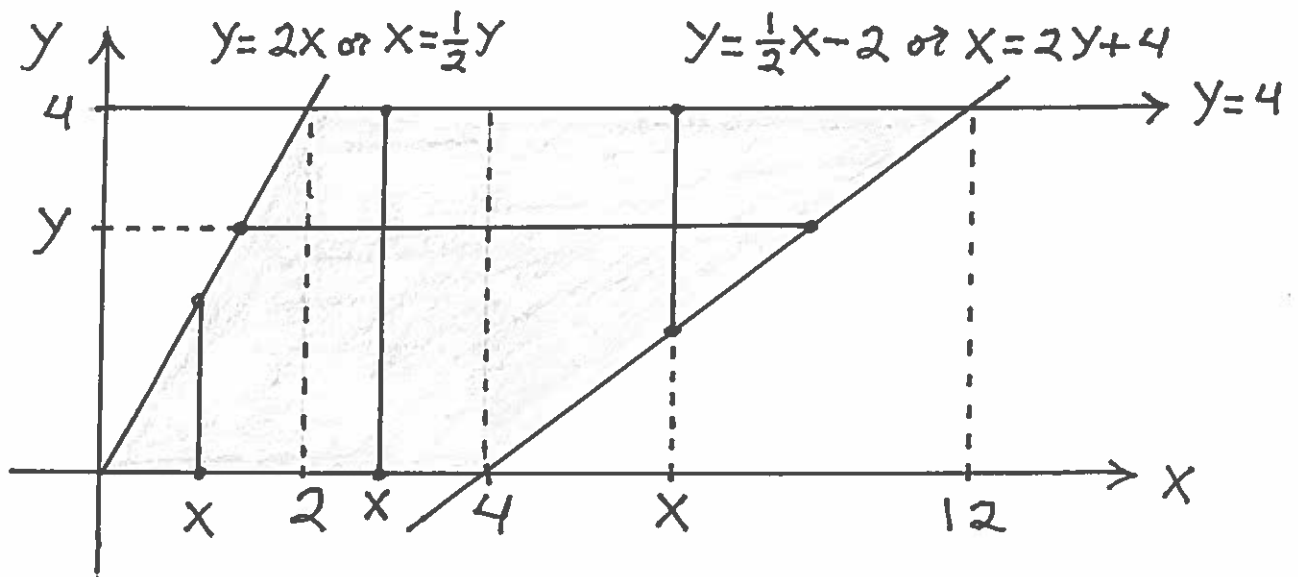
2.) $y = x^3, y = 8, x = -1$
 $y = x^3$ or $x = y^{1/3}$



a.) $AREA = \int_{-1}^2 (8 - x^3) dx$

b.) $AREA = \int_{-1}^8 (y^{1/3} - (-1)) dy$

3.) $y = 2x$, $y = \frac{1}{2}x - 2$, $y = 4$, $y = 0$



$$\begin{aligned}
 \text{a.) AREA} &= \int_0^{12} (\text{TOP} - \text{BOTTOM}) dx \\
 &= \int_0^2 (2x - 0) dx + \int_2^4 (4 - 0) dx \\
 &\quad + \int_4^{12} (4 - (\frac{1}{2}x - 2)) dy
 \end{aligned}$$

$$\begin{aligned}
 \text{b.) AREA} &= \int_0^4 (\text{RIGHT} - \text{LEFT}) dy \\
 &= \int_0^4 ((2y + 4) - \frac{1}{2}y) dy
 \end{aligned}$$