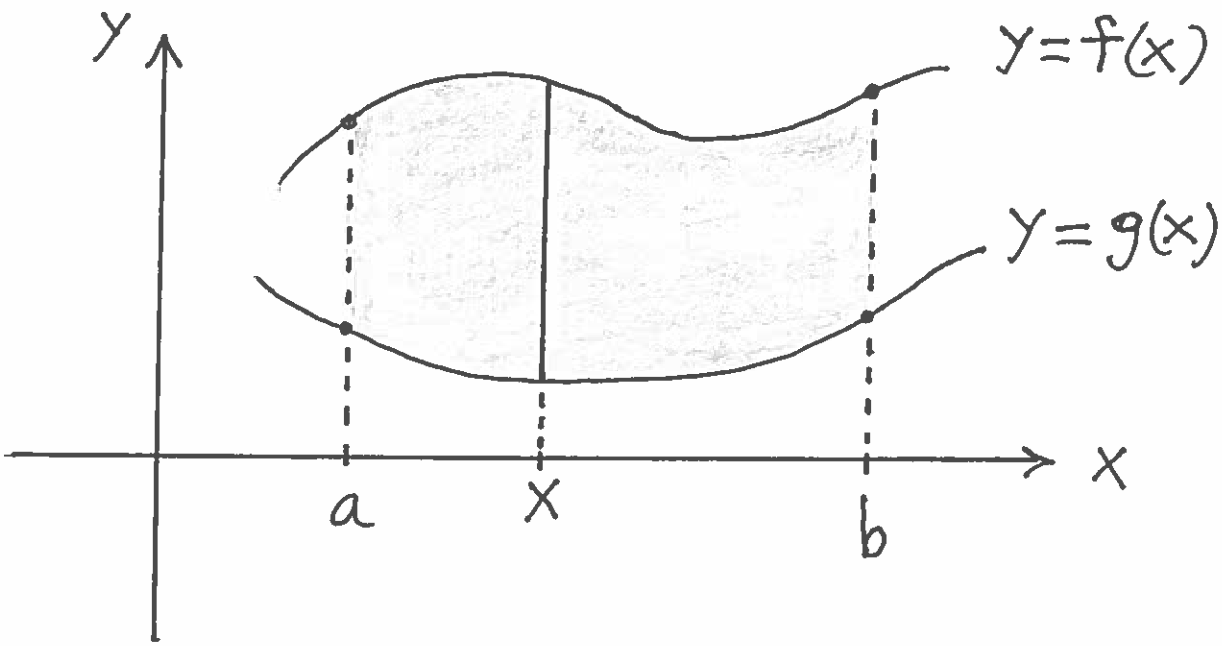
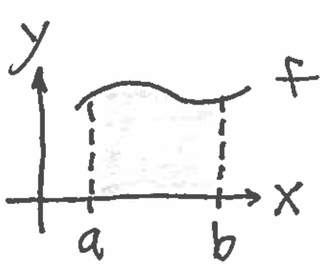
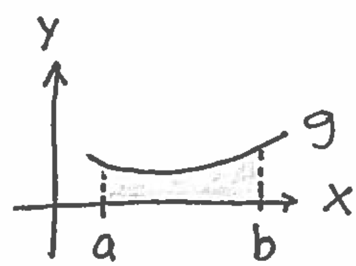
Math 16B
Section 5.5

Area Between Curves

Assume that $f(x) \geq g(x)$ for $a \leq x \leq b$. Find the AREA of the shaded region below:



AREA = 

 − 

(SEE next page.)

$$= \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$

$$= \int_a^b (f(x) - g(x))\,dx \text{, i.e.,}$$

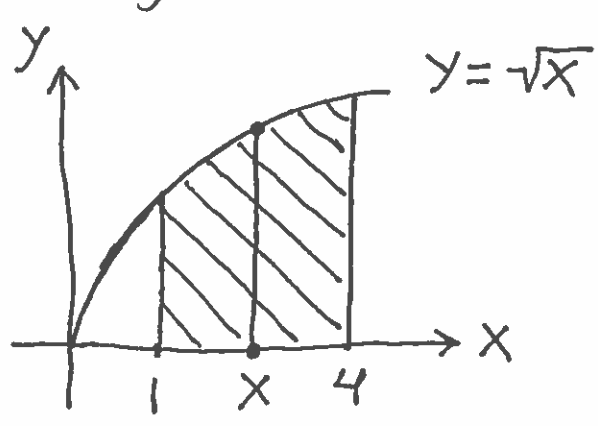
$$\boxed{\text{AREA} = \int_a^b (\underbrace{f(x)}_{\text{TOP}} - \underbrace{g(x)}_{\text{BOTTOM}})\,dx}$$

<u>Example</u>: Find the AREA of the region bounded by the graphs of the given equations (SET UP ONLY)

1.) $y = \sqrt{x}$, $y = 0$, $x = 1$, $x = 4$

$$\text{AREA} = \int_1^4 (\sqrt{x} - 0)\,dx$$

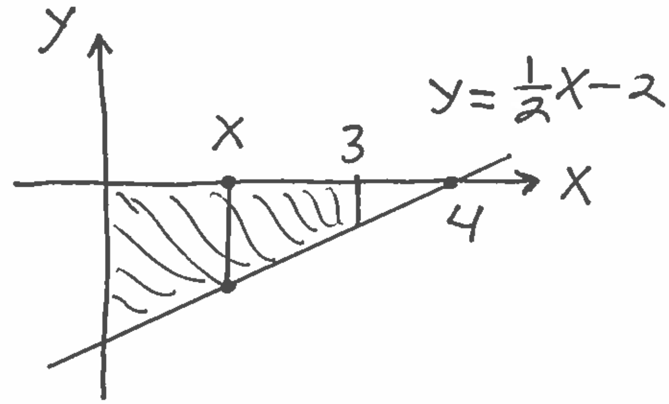
$$= \int_1^4 \sqrt{x}\,dx$$



2.) $y = \frac{1}{2}x - 2, \; y = 0, \; x = 0, \; x = 3$

$$\text{AREA} = \int_0^3 \left(0 - \left(\tfrac{1}{2}x - 2\right)\right) dx$$

$$= \int_0^3 \left(2 - \tfrac{1}{2}x\right) dx$$



3.) $y = x^2, \; y = x + 6$

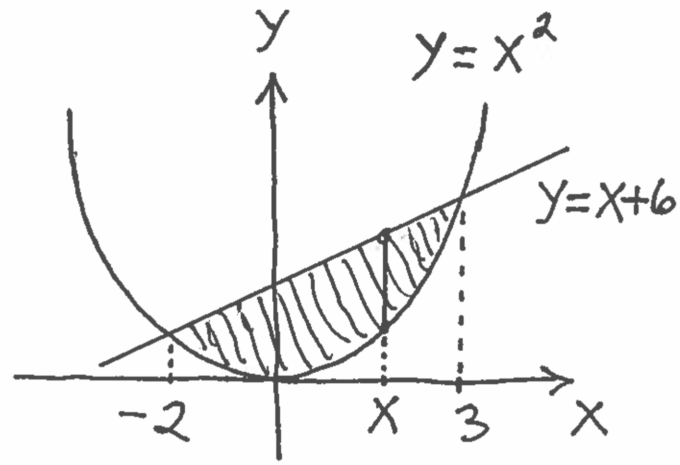
$x^2 = x + 6 \longrightarrow$

$x^2 - x - 6 = 0 \longrightarrow$

$(x-3)(x+2) = 0$

$\downarrow \qquad \searrow$

$x = 3 \qquad x = -2$

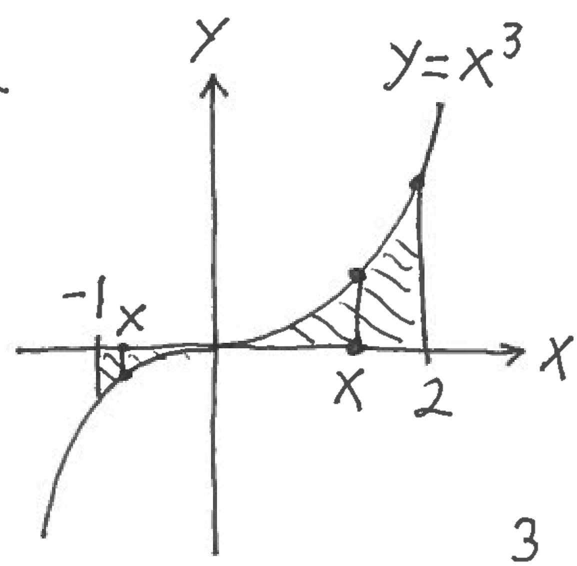


$$\text{AREA} = \int_{-2}^{3} \left[x + 6 - x^2\right] dx$$

4.) $y = x^3, \; y = 0, \; x = -1, \; x = 2$

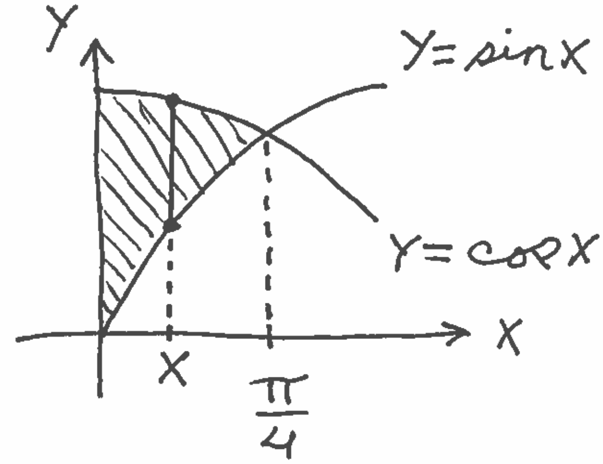
$$\text{AREA} = \int_{-1}^{0} (0 - x^3)\, dx$$

$$+ \int_0^2 (x^3 - 0)\, dx$$



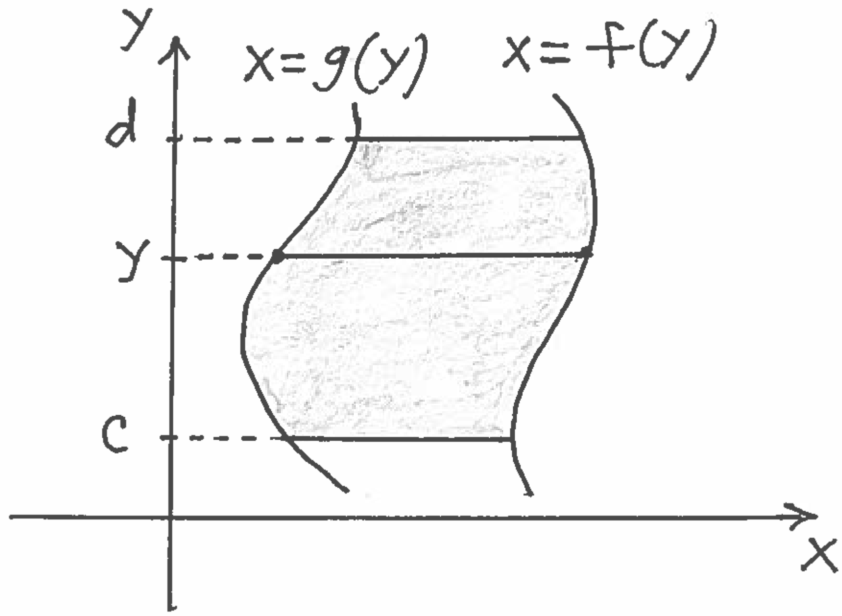
5.) $y = \sin x,\ y = \cos x,\ x = 0$

$$\text{AREA} = \int_0^{\frac{\pi}{4}} (\cos x - \sin x)\, dx$$



Variation of Area Between Curves

assume that $f(y) \geq g(y)$ for $c \leq y \leq d$.



area of shaded region is

$$\text{AREA} = \int_c^d f(y)\, dy - \int_c^d g(y)\, dy$$

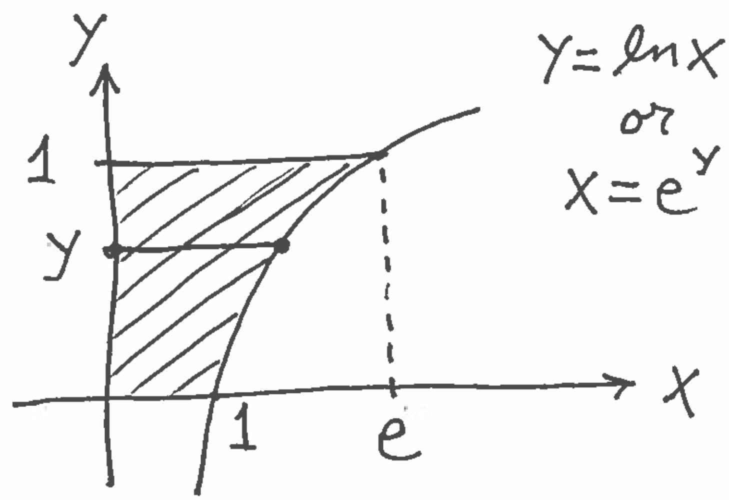
(SEE next page.)

$$= \int_c^d (f(y) - g(y))\, dy, \text{ i.e.,}$$

$$\boxed{\text{AREA} = \int_c^d (\underbrace{f(y)}_{\text{RIGHT}} - \underbrace{g(y)}_{\text{LEFT}})\, dy}$$

6.) $y = \ln x$, $y = 0$, $y = 1$, $x = 0$

$$\text{AREA} = \int_0^1 (e^y - 0)\, dy$$



7.) $x = 2 - y$, $x = y^2$

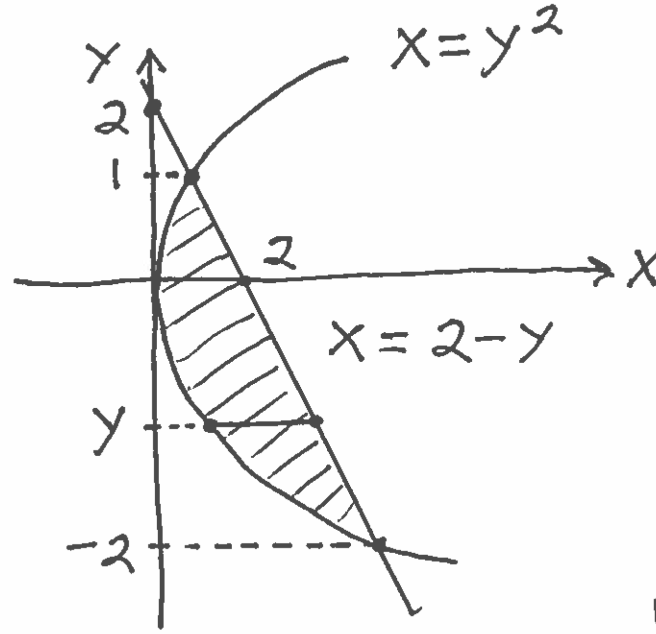
$y^2 = 2 - y \rightarrow$

$y^2 + y - 2 = 0 \rightarrow$

$(y-1)(y+2) = 0 \rightarrow y = 1,$

$y = -2$

$$\text{AREA} = \int_{-2}^1 ((2-y) - y^2)\, dy$$

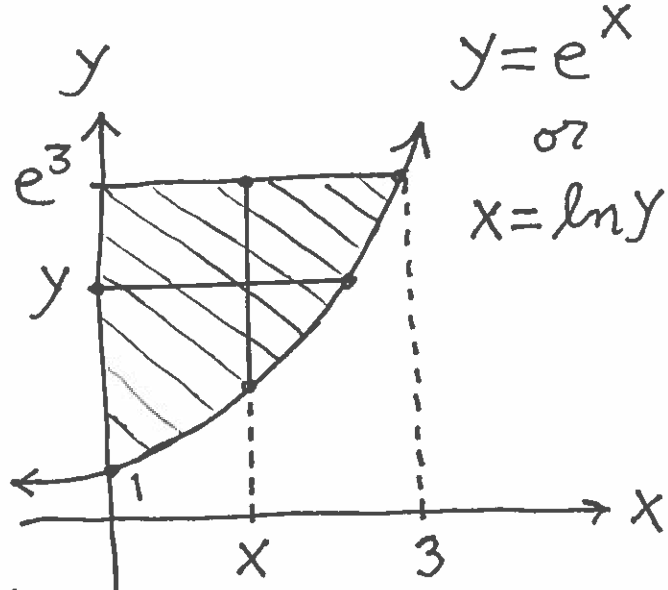


Example: Find the AREA of the region bounded by the graphs of the given equations (SET UP ONLY) using

a.) VERTICAL cross-sections.

b.) HORIZONTAL cross-sections.

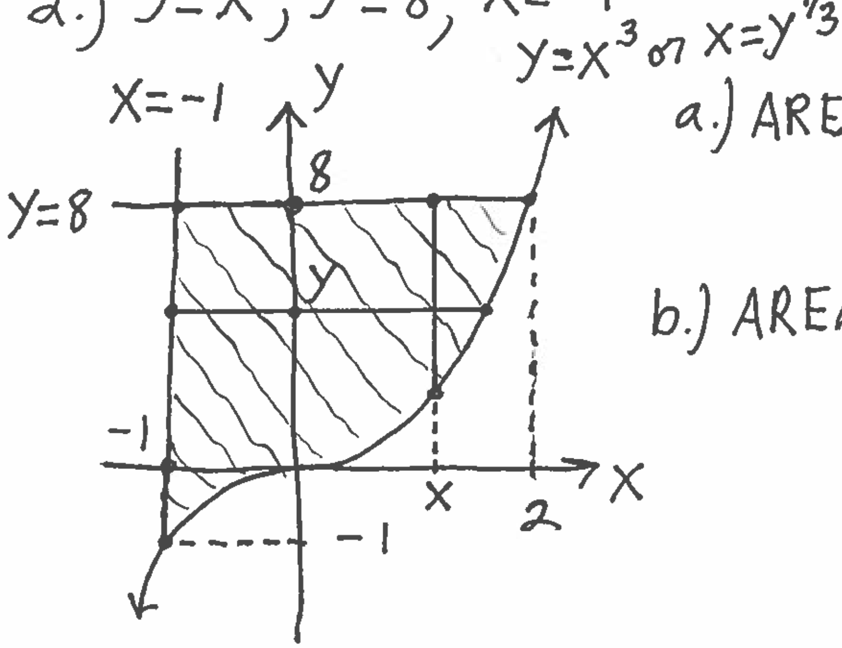
1.) $y=e^x$, $y=e^3$, $x=0$



a.) $\text{AREA} = \int_0^3 (e^3 - e^x)\,dx$

b.) $\text{AREA} = \int_1^{e^3} (\ln y - 0)\,dy$

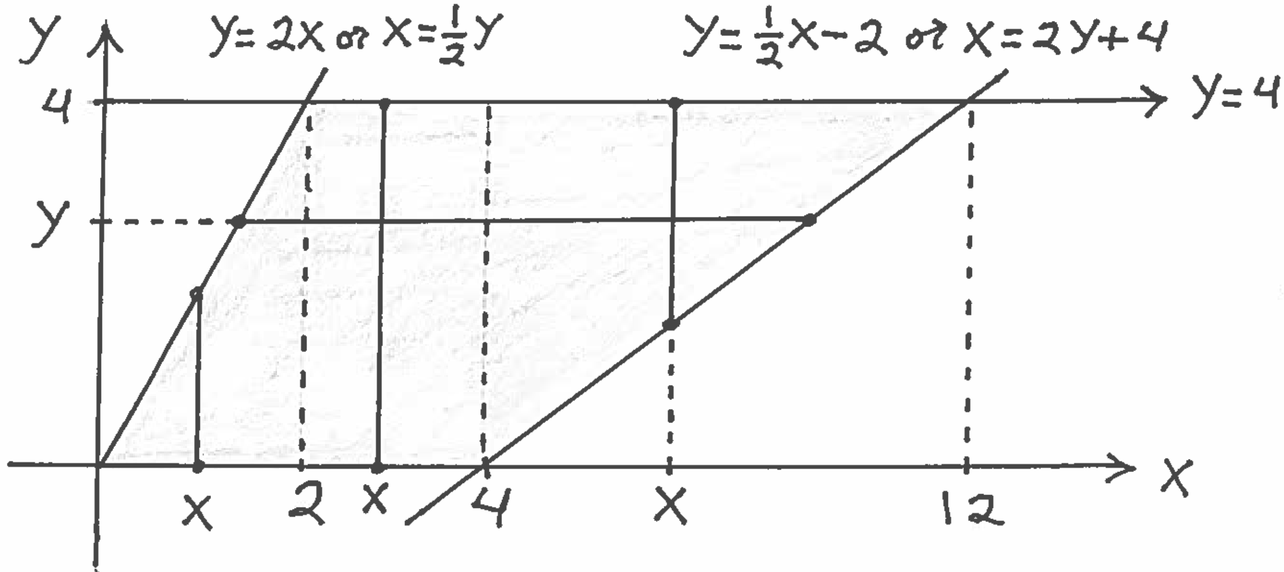
2.) $y=x^3$, $y=8$, $x=-1$



a.) $\text{AREA} = \int_{-1}^{2} (8 - x^3)\,dx$

b.) $\text{AREA} = \int_{-1}^{8} (y^{1/3} - (-1))\,dy$

3.) $y = 2x,\ y = \frac{1}{2}x - 2,\ y = 4,\ y = 0$



a.) $\text{AREA} = \int_0^{12} (\text{TOP} - \text{BOTTOM})\, dx$

$$= \int_0^2 (2x - 0)\, dx + \int_2^4 (4 - 0)\, dx$$

$$+ \int_4^{12} \left(4 - \left(\tfrac{1}{2}x - 2\right)\right) dy$$

b.) $\text{AREA} = \int_0^4 (\text{RIGHT} - \text{LEFT})\, dy$

$$= \int_0^4 \left((2y + 4) - \tfrac{1}{2}y\right) dy$$