Math 16B
Section 5.7

Disc Method – Finding the Volume of a Solid of Revolution

FACT: $\int_a^b A(x)\,dx = \text{Volume}$

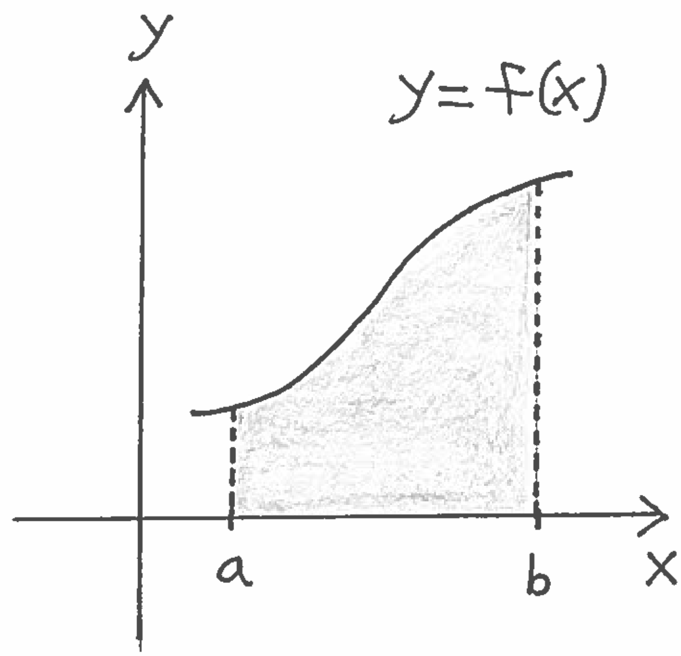
$A(x)$: cross-sectional area

dx: width

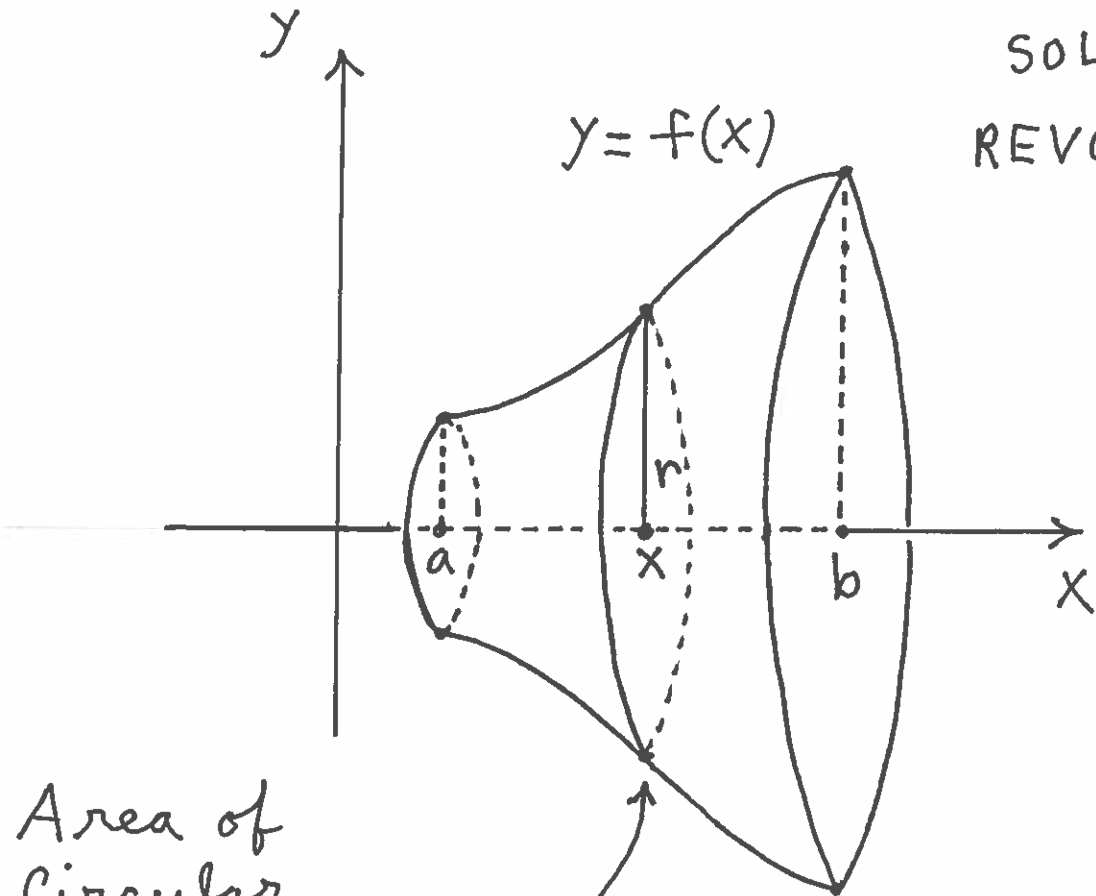
Consider region R given in the diagram. Create a

SOLID OF REVOLUTION

by revolving R about the x-axis.



SOLID OF REVOLUTION



Area of circular slice at x is

$$A(x) = \pi r^2 = \pi\,(f(x))^2 \, ;$$

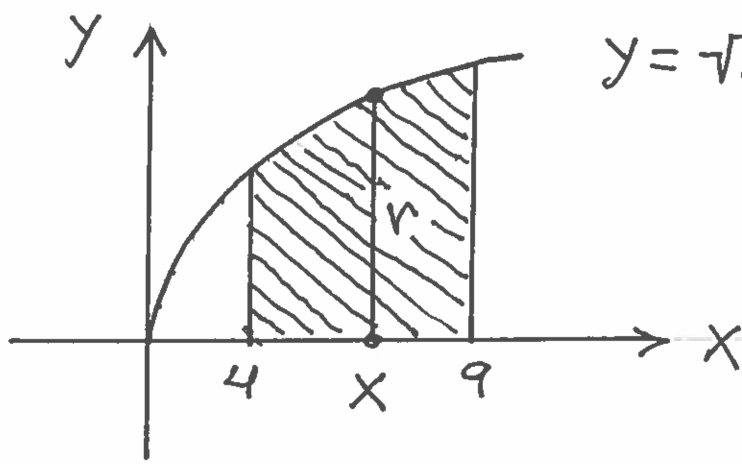
Thus the Volume of the Solid of Revolution is

$$\left.\begin{aligned} \text{Volume} &= \int_a^b A(x)\,dx \\ &= \pi \int_a^b (f(x))^2\,dx \end{aligned}\right\} \quad \text{DISC METHOD}$$

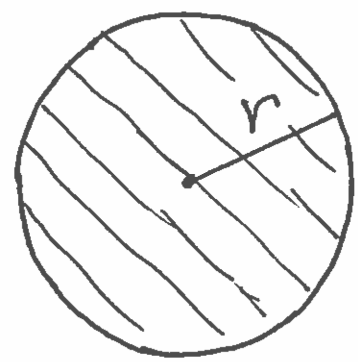
radius of circular slice

Example: Find the volume of the solid formed by revolving region R, which is bounded by the graphs of the given equations, about the x-axis. (SET UP ONLY!)

1.) $y=\sqrt{x}$, $y=0$, $x=4$, and $x=9$:



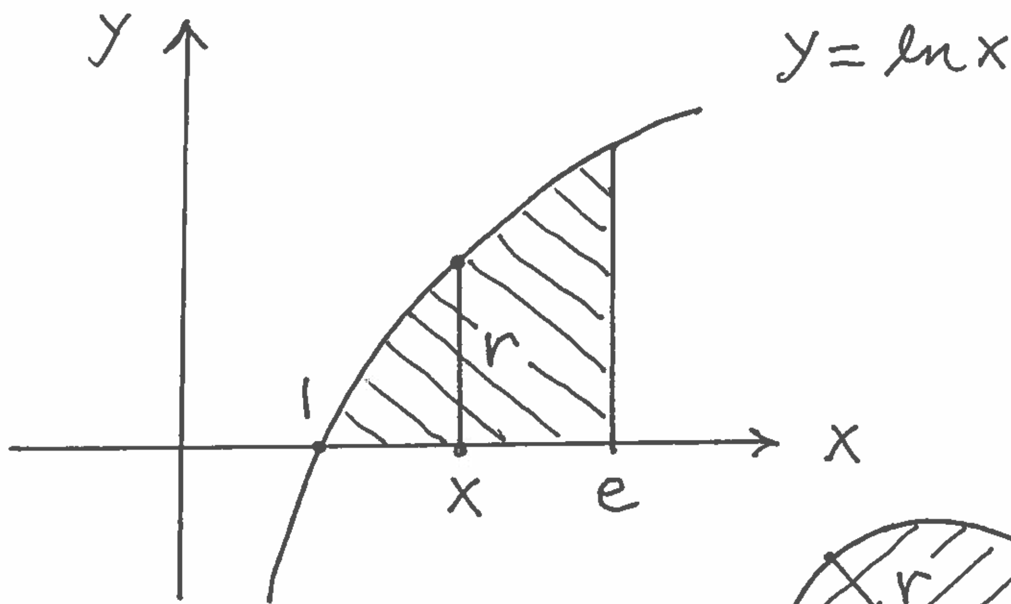
Circular slice at x has area

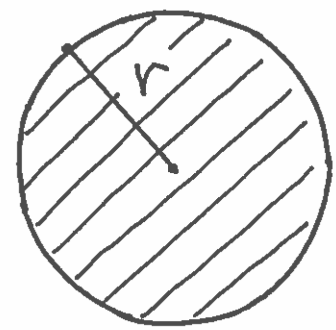


$A(x) = \pi r^2 = \pi(\sqrt{x})^2$, so

$$\text{Volume} = \pi \int_4^9 (\sqrt{x})^2 \, dx$$

2.) $y=\ln x$, $y=0$, and $x=e$:



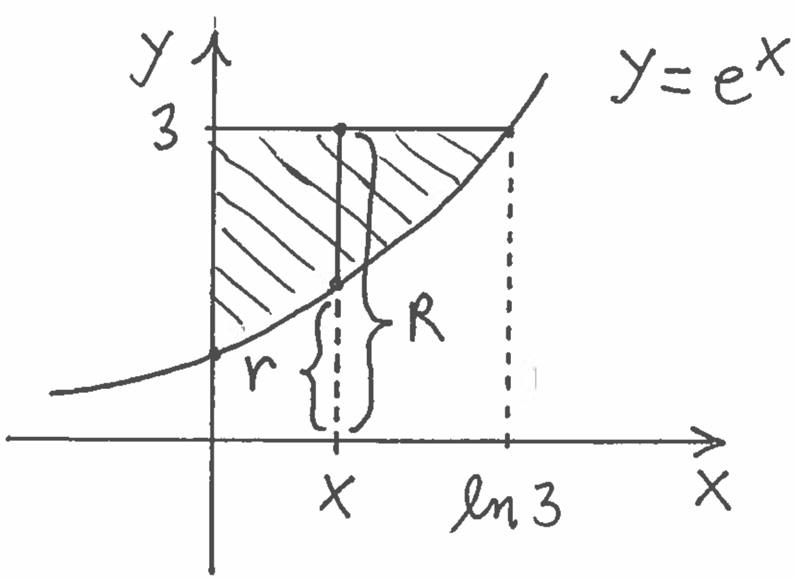


Circular slice at x has area

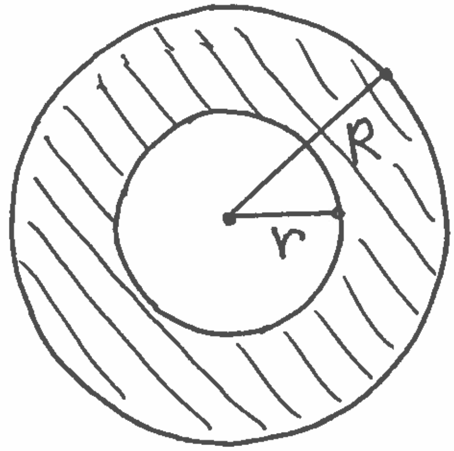
$$A(x) = \pi r^2 = \pi (\ln x)^2, \text{ so}$$

$$\text{Volume} = \pi \int_1^e (\ln x)^2 \, dx$$

(*) 3.) $y = e^x$, $x = 0$, and $y = 3$:



Circular slice at x
has area



$$A(x) = \pi R^2 - \pi r^2$$

$$= \pi(3)^2 - \pi(e^x)^2, \text{ so}$$

$$\text{Volume} = \int_0^{\ln 3} [\pi(3)^2 - \pi(e^x)^2]\, dx$$

We will now generalize the Disc Method by revolving regions R about the y-axis, any vertical line, and any horizontal line.

Math 16B
Kouba
Summary of Finding Volumes of Solids of Revolution

1.) Make a LARGE drawing of the flat region R , clearly labeling the equations and points of intersection.

2.) Spin region R about the given axis and visualize in your mind's eye what the solid looks like. A rough sketch of the solid is helpful.

3.) Decide whether to slice the solid vertically at x or horizontally at y in order to create circular cross-sections.

4.) Determine if the circular slice is a solid circle of radius r and Area = πr^2 , or a circle with a whole in it (donut) of Area = $\pi R^2 - \pi r^2$.

5.) Clearly mark r and/or R on your original graph together with either x or y.

6.) Write r or r and R as equations in either variable x or variable y.

7.) Finally, set up the equation for Volume by integrating the Area Equation πr^2 or $\pi R^2 - \pi r^2$ using dx or dy, the appropriate interval, and your equations from step 6.

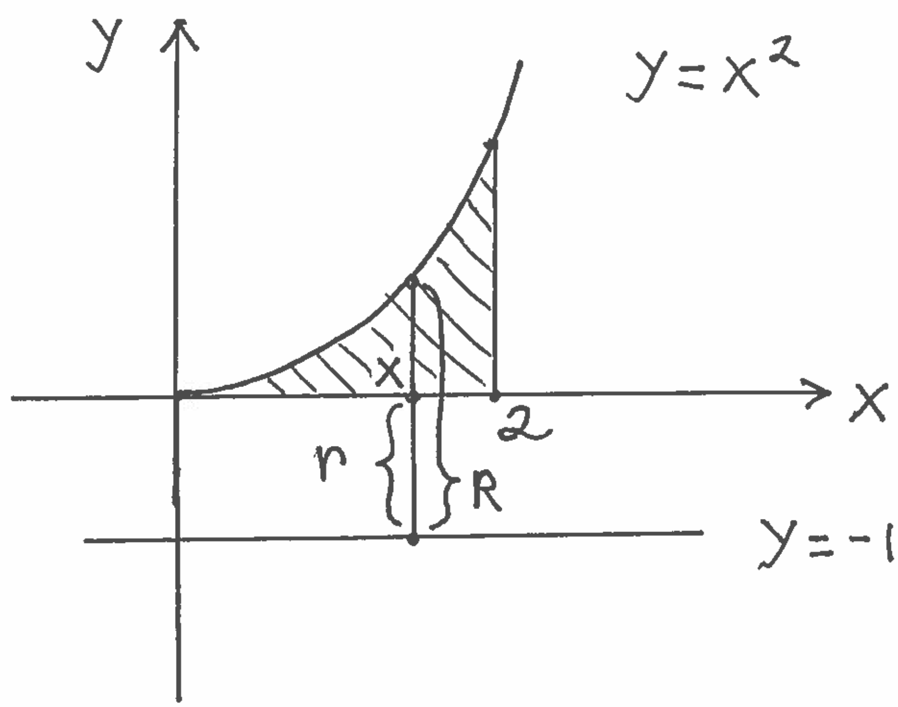
Example: Find the volume of the solid formed by revolving the given region, which is bounded by the graphs of the given equations, about the given line.

1.) $y = x^2$, $x = 2$, and $y = 0$:

a.) the line $y = -1$

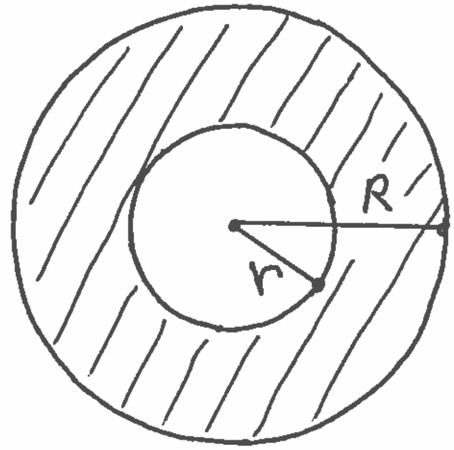
b.) the line $y = 5$

a.)



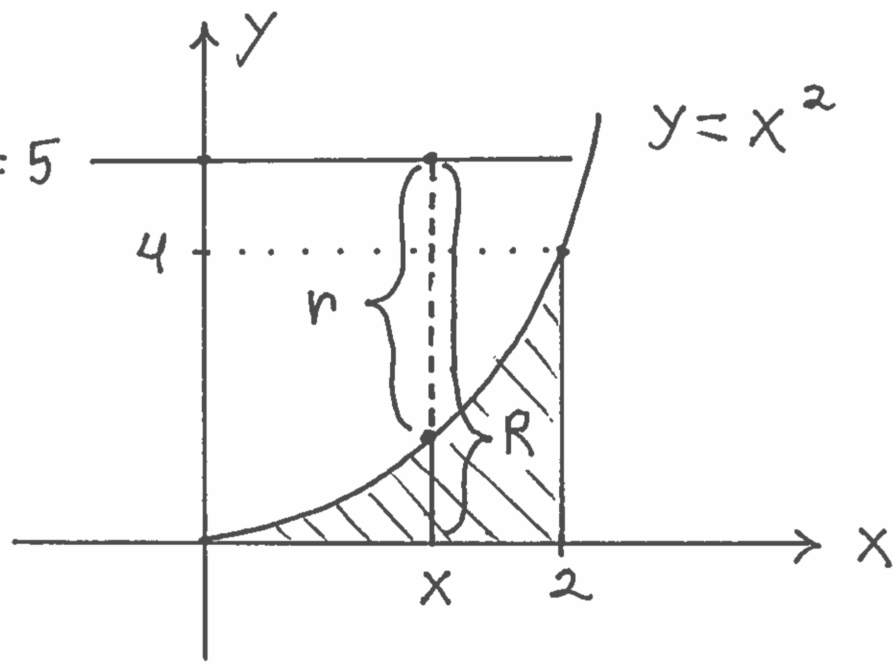
The circular slice at x has area

$$A(x) = \pi R^2 - \pi r^2$$
$$= \pi (x^2+1)^2 - \pi (1)^2, \text{ so}$$



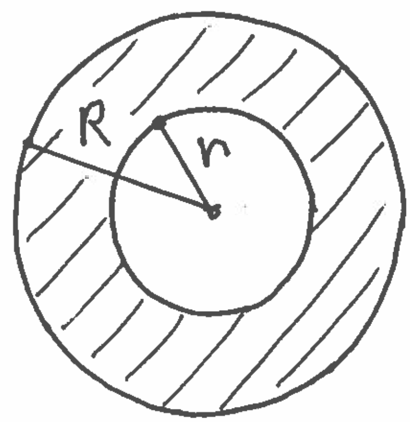
$$\text{Volume} = \int_0^2 \left[\pi (x^2+1)^2 - \pi (1)^2\right] dx$$

b.)



The circular slice at x has area

$$A(x) = \pi R^2 - \pi r^2$$
$$= \pi (5)^2 - \pi (5 - x^2)^2, \text{ so}$$



$$\text{Volume} = \int_0^2 \left[\pi(5)^2 - \pi(5-x^2)^2\right] dx$$

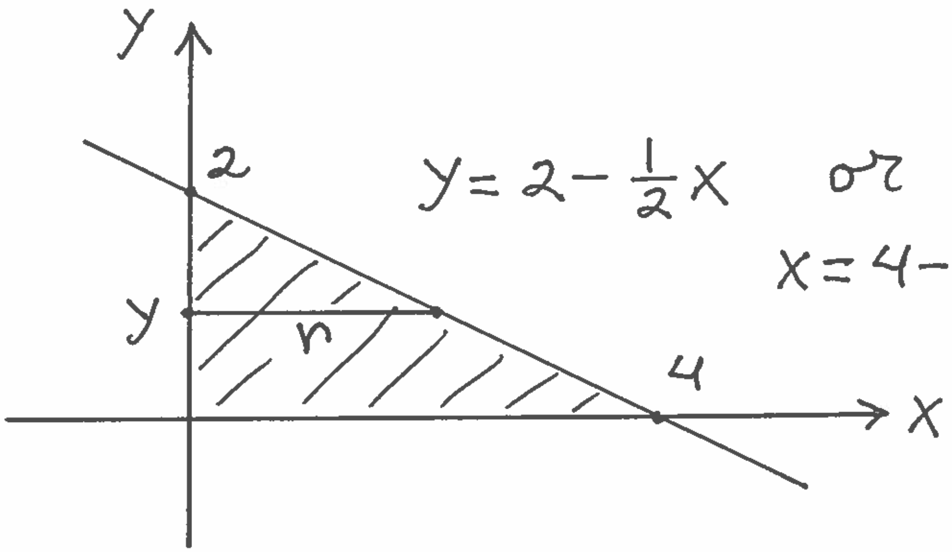
2.) $y = 2 - \frac{1}{2}x$, $x = 0$, and $y = 0$:

a.) the y-axis

b.) the line $x = -3$

c.) the line $x = 6$

a.)

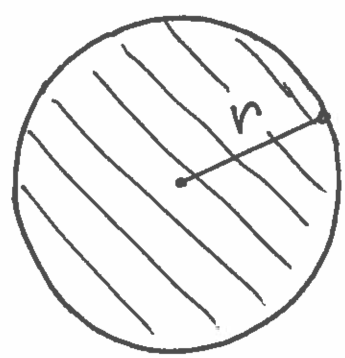


$y = 2 - \frac{1}{2}x$ or

$x = 4 - 2y$

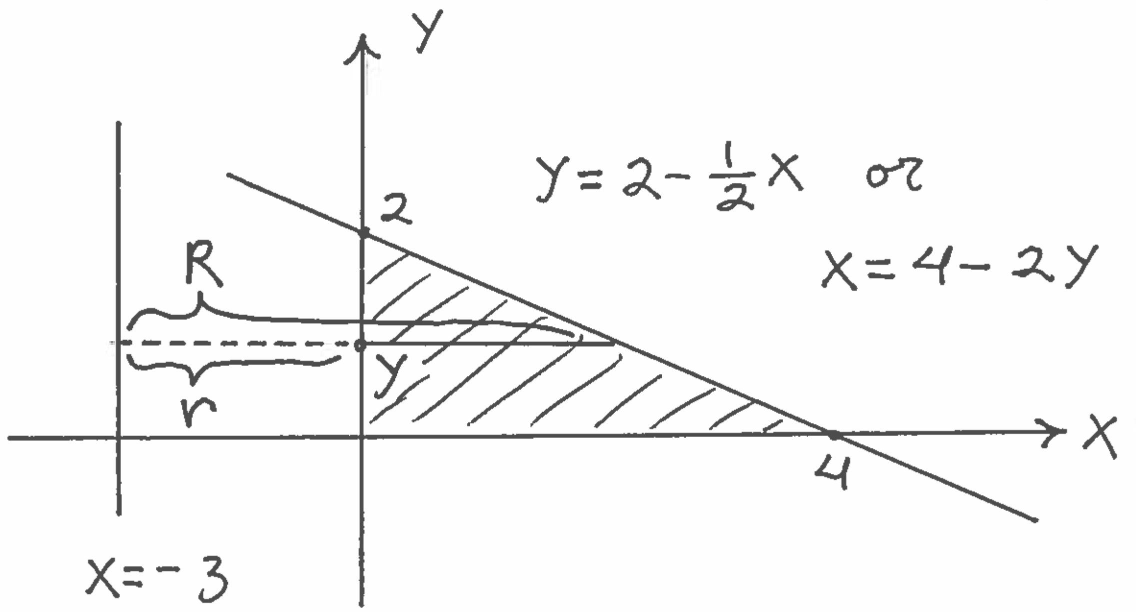
The circular slice at y has area

$A(y) = \pi r^2 = \pi(4-2y)^2$, so

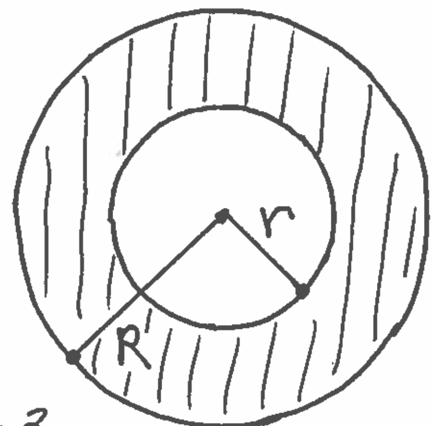


$$\text{Volume} = \int_0^2 \pi(4-2y)^2 \, dy$$

b.)



The circular slice at y has area

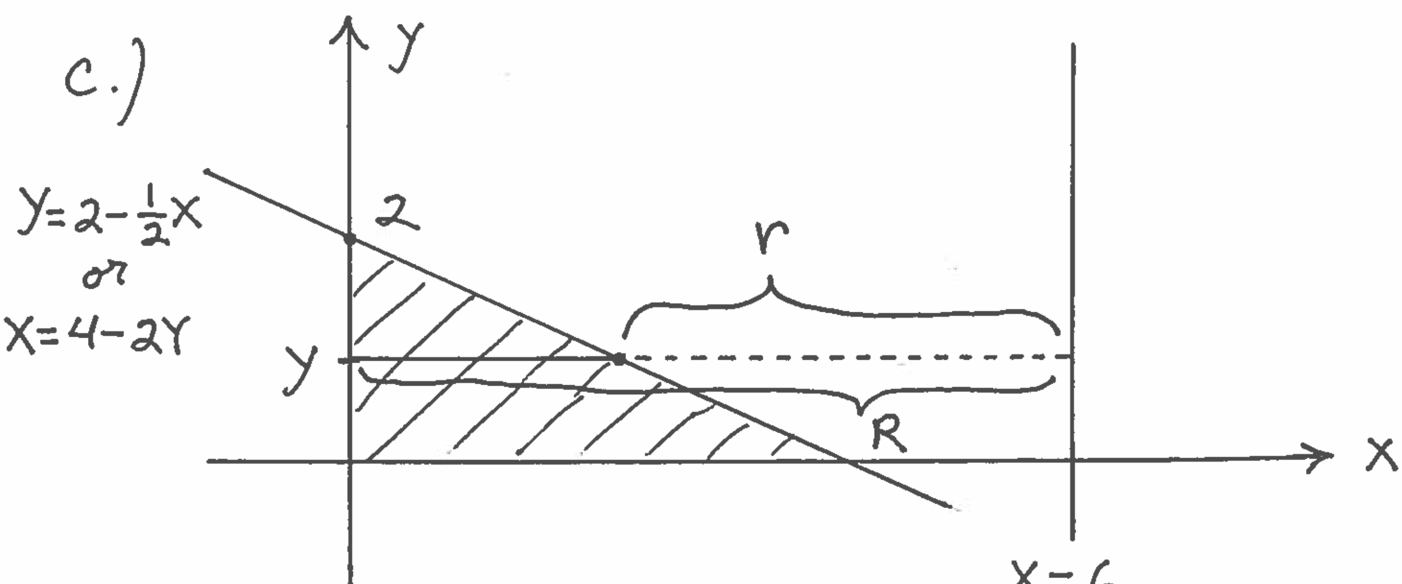


$$A(y) = \pi R^2 - \pi r^2$$

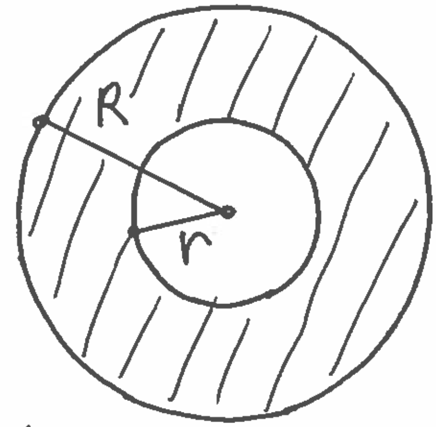
$$= \pi\left((4-2y)+3\right)^2 - \pi(3)^2, \text{ so}$$

$$\text{Volume} = \int_0^2 \left[\pi\left((4-2y)+3\right)^2 - \pi(3)^2\right] dy$$

c.)



The circular slice at y has area



$$A(y) = \pi R^2 - \pi r^2$$

$$= \pi(6)^2 - \pi(6-(4-2y))^2, \text{ so}$$

$$\text{Volume} = \int_0^2 [\pi(6)^2 - \pi(6-(4-2y))^2]\, dy$$