Math 16B
Section 6.1

More U- Substitution

I.) Standard U- Substitution

1.) $\int \frac{x^2+2x+1}{x^3+3x^2+3x+7}\,dx$

(Let $u = x^3+3x^2+3x+7 \xrightarrow{D}$
$du = (3x^2+6x+3)\,dx = 3(x^2+2x+1)\,dx \rightarrow$
$\frac{1}{3}du = (x^2+2x+1)\,dx$)

$= \frac{1}{3}\int \frac{1}{u}\,du = \frac{1}{3}\ln|u| + C$

$= \frac{1}{3}\ln|x^3+3x^2+3x+7| + C$

2.) $\int x e^{x^2}\cos(e^{x^2})\sin(e^{x^2})\,dx$

(Let $u = \sin(e^{x^2}) \xrightarrow{D}$
$du = \cos(e^{x^2})\cdot e^{x^2}\cdot 2x\,dx \rightarrow$
$\frac{1}{2}du = x e^{x^2}\cos(e^{x^2})\,dx$)

II.) U-Substitution with a "Back" Substitute

3.) $\int \frac{x}{x+4}\,dx$ (Let $u = x+4 \xrightarrow{D}$ $du = 1\,dx$ AND $x = u-4$)

$= \int \frac{u-4}{u}\,du = \int \left[\frac{u}{u} - \frac{4}{u}\right] du$

$= \int \left[1 - 4\cdot\frac{1}{u}\right] du$

$= u - 4\ln|u| + C$

$= (x+4) - 4\ln|x+4| + C$

4.) $\int (x-1)\sqrt{x+1}\,dx$ (Let $u = x+1 \xrightarrow{D}$ $du = 1\,dx$ AND $x = u-1$)

$= \int ((u-1)-1)\sqrt{u}\,du = \int (u-2)\,u^{1/2}\,du$

$= \int (u^{3/2} - 2u^{1/2})\,du$

$= \frac{2}{5}u^{5/2} - 2\cdot\frac{2}{3}u^{3/2} + C$

$$= \frac{2}{5}(x+1)^{5/2} - \frac{4}{3}(x+1)^{3/2} + c$$

5.) $\int x^3 (x^2+1)^7 \, dx$

$$= \int x^2 \cdot x \cdot (x^2+1)^7 \, dx$$

(Let $u = x^2+1 \xrightarrow{D} du = 2x\,dx \longrightarrow$

$\frac{1}{2} du = x\,dx$ AND $x^2 = u-1$)

$$= \frac{1}{2}\int (u-1)\,u^7 \, du$$

$$= \frac{1}{2}\int (u^8 - u^7)\, du$$

$$= \frac{1}{2}\left(\frac{1}{9}u^9 - \frac{1}{8}u^8\right) + c$$

$$= \frac{1}{2}\left(\frac{1}{9}(x^2+1)^9 - \frac{1}{8}(x^2+1)^8\right) + c$$

III.) Power U-Substitution

6.) $\displaystyle\int \frac{\sqrt{x}}{1+\sqrt{x}}\,dx$

(Let $x = u^2 \xrightarrow{D} dx = 2u\,du$) AND $u = \sqrt{x}$

$$= \int \frac{\sqrt{u^2}}{1+\sqrt{u^2}} \cdot 2u\,du$$

$$= \int \frac{u}{1+u} \cdot 2u\,du$$

$$= \int \frac{2u^2}{1+u}\,du$$

$$= \int \left[2u - 2 + \frac{2}{u+1}\right] du$$

$$\begin{array}{r} 2u - 2 \\ u+1 \overline{\big) 2u^2 } \\ -(2u^2+2u) \\ \hline -2u \\ -(-2u-2) \\ \hline 2 \end{array}$$

$$= u^2 - 2u + 2\ln|u+1| + C$$

$$= x - 2\sqrt{x} + 2\ln|\sqrt{x}+1| + C$$

7.) $\displaystyle\int \frac{1}{2+x^{1/3}}\,dx$

$\left(\text{Let } x = u^3 \xrightarrow{D} dx = 3u^2\,du\right)$

AND $u = x^{1/3}$

$$= \int \frac{1}{2+(u^3)^{1/3}} \cdot 3u^2\,du$$

$$= \int \frac{3u^2}{2+u}\,du$$

$$= \int \left[3u - 6 + \frac{12}{u+2}\right] du$$

$$\begin{array}{r} 3u-6 \\ u+2\,\overline{)\,3u^2 \qquad\qquad} \\ -\underline{(3u^2+6u)} \\ -6u \qquad \\ -\underline{(-6u-12)} \\ 12 \end{array}$$

$$= \frac{3}{2}u^2 - 6u + 12\ln|u+2| + C$$

$$= \frac{3}{2}(x^{1/3})^2 - 6x^{1/3} + 12\ln|x^{1/3}+2| + C$$