Math 16B
Section 6.2

Integration by Parts

Let's think of Integration by Parts as the "reverse" of the Product Rule. See its derivation on the next page.

Math 16B
Kouba
Integration by Parts Formula

__*RECALL*__ : (Product Rule) $D\{f(x)g(x)\} = f(x)g'(x) + f'(x)g(x) \quad \longrightarrow$

$$f(x)g'(x) = D\{f(x)g(x)\} - f'(x)g(x) \quad \longrightarrow$$

$$f(x)g'(x) = D\{f(x)g(x)\} - g(x)f'(x) \quad \longrightarrow$$

$$\int f(x)g'(x)\ dx = \int D\{f(x)g(x)\}\ dx - \int g(x)f'(x)\ dx \quad \longrightarrow$$

$$\int f(x)g'(x)\ dx = f(x)g(x) - \int g(x)f'(x)dx \quad \longrightarrow$$

$$\left(\text{Let } u = f(x) \longrightarrow^{D} du = f'(x)\ dx \ \text{ and } \ \text{let } v = g(x) \longrightarrow^{D} dv = g'(x)\ dx\right) \quad \longrightarrow$$

(*INTEGRATION by PARTS*)

$$\int u\ dv = uv - \int v\ du$$

2

<u>Example</u>: Use Integration by Parts for each antiderivative.

1.) $\int x \sin x \, dx$

(Let $u = x$, $dv = \sin x \, dx$

$\rightarrow du = 1\,dx$, $v = -\cos x + \cancelto{0}{c}$;

always choose constant $c = 0$!)

$= x(-\cos x) - {}^{-}\int \cos x \, dx$

$= -x\cos x + \sin x + c$

2.) $\int x e^{3x} \, dx$

(Let $u = x$, $dv = e^{3x} \, dx$

$\longrightarrow du = 1\,dx$, $v = \frac{1}{3} e^{3x}$)

$= x \cdot \frac{1}{3} e^{3x} - \frac{1}{3} \int e^{3x} \, dx$

$= \frac{1}{3} x e^{3x} - \frac{1}{3} \cdot \frac{1}{3} e^{3x} + c$

3.) $\int_1^e x^2 \ln x \, dx$

(Let $u = \ln x$, $dv = x^2 dx$

$\rightarrow du = \frac{1}{x} dx$, $v = \frac{1}{3}x^3$)

$$= \frac{1}{3}x^3 \ln x \Big|_1^e - \frac{1}{3}\int_1^e \frac{1}{x} x^3 \, dx$$

$$= \frac{1}{3}e^3 \cancelto{1}{\ln e} - \frac{1}{3}1^3 \cancelto{0}{\ln 1} - \frac{1}{3}\int_1^e x^2 \, dx$$

$$= \frac{1}{3}e^3 - \frac{1}{3} \cdot \frac{1}{3}x^3 \Big|_1^e$$

$$= \frac{1}{3}e^3 - \frac{1}{9}\left(e^3 - 1^3\right)$$

$$= \frac{3}{9}e^3 - \frac{1}{9}e^3 + \frac{1}{9}$$

$$= \frac{2}{9}e^3 + \frac{1}{9}$$

(*) 4.) $\int x^3 (1+x^2)^5 \, dx$

$= \int x^2 \cdot x (1+x^2)^5 \, dx$

(Let $u = x^2$, $dv = x(1+x^2)^5 dx$

$\rightarrow du = 2x \, dx$, $v = \frac{1}{6} \cdot \frac{1}{2} (1+x^2)^6$)

$= \frac{1}{12} x^2 (1+x^2)^6 - 2 \cdot \frac{1}{12} \int x (1+x^2)^6 dx$

$= \frac{1}{12} x^2 (1+x^2)^6 - \frac{1}{6} \cdot \frac{1}{7} \cdot \frac{1}{2} (1+x^2)^7 + C$

$= \frac{1}{12} x^2 (1+x^2)^6 - \frac{1}{84} (1+x^2)^7 + C$