Math 16B
Sections 4.1, 4.2

Rules for Exponents and Exponential Functions

The Number e

THE NUMBER e (attributed to Leonhard Euler, Swiss Mathematician (1707-1783))

EXAMPLE: Evaluate $(1+1/k)^k$ for the following values of k.

k	$(1+1/k)^k$
1	$2.00000\cdots$
10	$2.59374\cdots$
100	$2.70841\cdots$
1000	$2.71692\cdots$
10,000	$2.71814\cdots$
100,000	$2.71826\cdots$
1,000,000	$2.71828\cdots$
10,000,000	$2.71828\cdots$

CONCLUSION: $\lim\limits_{k\to\infty}(1+1/k)^k = e \approx 2.71828.$

Rules for Exponents

RECALL: The Rules for Exponents

1.) $x^m \cdot x^n = x^{m+n}$

2.) $(x^m)^n = x^{mn}$

3.) $(xy)^m = x^m y^m$

4.) $\frac{x^m}{x^n} = x^{m-n}$

5.) $\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$

6.) $x^{-m} = 1/x^m$

Example: Use the Rules of Exponents to simplify each expression.

1.) $\frac{7^{11}}{7^9} = 7^{11-9} = 7^2 = 49$

2.) $3^{-\frac{1}{2}} 3^{\frac{7}{2}} = 3^{-\frac{1}{2}+\frac{7}{2}} = 3^{\frac{6}{2}} = 3^3 = 27$

3.) $(x^{-3})^4 = x^{(-3)(4)} = x^{-12} = \frac{1}{x^{12}}$

4.) $$\frac{(e^3)^5}{e^2e^4} = \frac{e^{15}}{e^6} = e^9$$

5.) $$\left(\frac{x^{-1}}{x^{-5}}\right)^{1/2} = \left(x^{-1-(-5)}\right)^{1/2}$$

$$= \left(x^4\right)^{1/2} = x^2$$

6.) $$\left(\frac{x^{1/3}x^{1/2}}{x}\right)^{12} = \left(\frac{x^{\frac{2}{6}+\frac{3}{6}}}{x^1}\right)^{12}$$

$$= \left(\frac{x^{5/6}}{x^{6/6}}\right)^{12} = \left(x^{-1/6}\right)^{12} = x^{-2} = \frac{1}{x^2}$$

7.) $$\left(\frac{(e^{-3})^2}{e^2 e^{-4}}\right)^{-5} = \left(\frac{e^{-6}}{e^{-2}}\right)^{-5}$$

$$= \left(e^{-6-(-2)}\right)^{-5} = \left(e^{-4}\right)^{-5} = e^{20}$$

Avoid These Common Mistakes

1.) $(a+b)^m = a^m + b^m$

2.) $a^m a^n = a^{mn}$

3.) $\dfrac{a^m}{a^n} = a^{m/n}$

4.) $\dfrac{1}{a^{-m} + b^{-n}} = a^m + b^n$

5.) $\dfrac{a}{b+c} = \dfrac{a}{b} + \dfrac{a}{c}$

6.) $a^m + a^n = a^{m+n}$

FACT: If $a^m = a^n$, then $m = n$.

Example: Solve for t.

1.) $e^{3t+7} = e^2 e^t \longrightarrow$

$e^{3t+7} = e^{2+t} \longrightarrow 3t+7 = 2+t$

$\longrightarrow 2t = -5 \longrightarrow t = -5/2$

2.) $64^t = 2 \cdot 4^{t+1} \longrightarrow$

(Fake Math: $2 \cdot 4^{t+1} = 8^{t+1}$!!!)

$(2^6)^t = 2 \cdot (2^2)^{t+1} \longrightarrow$

$2^{6t} = 2^1 \cdot 2^{2t+2} \longrightarrow$

$2^{6t} = 2^{2t+3} \longrightarrow 6t = 2t+3 \longrightarrow$

$4t = 3 \longrightarrow t = 3/4$

3.) $9^{t+1} = \left(\frac{1}{3}\right)^{2-t} \longrightarrow$

$$(3^2)^{t+1} = (3^{-1})^{2-t} \longrightarrow$$

$$3^{2t+2} = 3^{-2+t} \longrightarrow$$

$$2t+2 = -2+t \rightarrow \quad t = -4$$

4.) $$\left(\frac{1}{125}\right)^{t-2} = 5 \cdot 5^{2t^2+3} \longrightarrow$$

$$(5^{-3})^{t-2} = 5^1 5^{2t^2+3} \longrightarrow$$

$$5^{-3t+6} = 5^{2t^2+4} \longrightarrow$$

$$-3t+6 = 2t^2+4 \longrightarrow$$

$$0 = 2t^2+3t-2 \longrightarrow$$

$$0 = (2t-1)(t+2) \longrightarrow$$

$$t = \frac{1}{2}, \quad t = -2$$

(Optional Practice Problems)

5.) $5^3 5^{t-1} = \dfrac{5^{2t+1}}{5^4}$

ANS: $t = 5$

6.) $16^{-t} = 4 \cdot 8^{1+t}$

ANS: $t = -5/7$

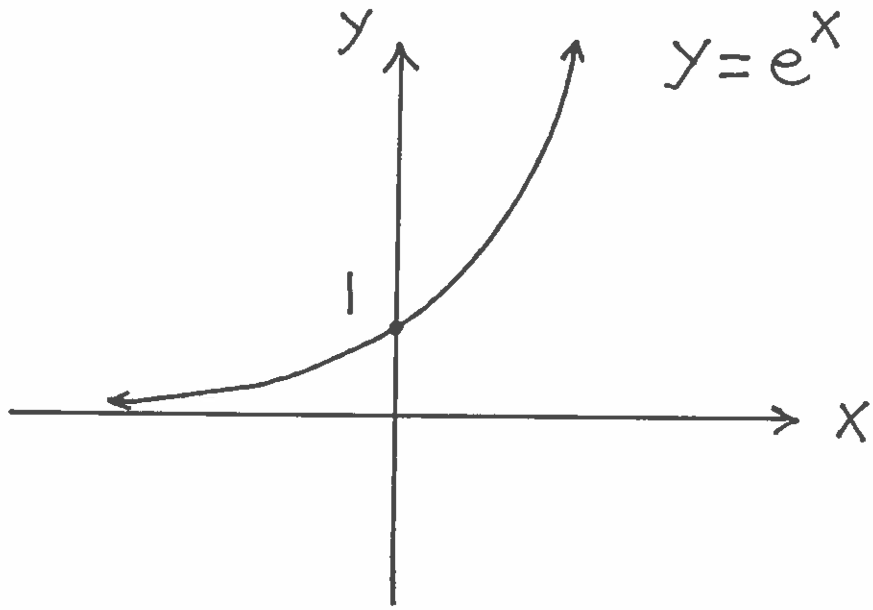
7.) $\left(\frac{1}{27}\right)^{t-2} = 3 \cdot 9^{t+2}$

ANS: $t = 1/5$

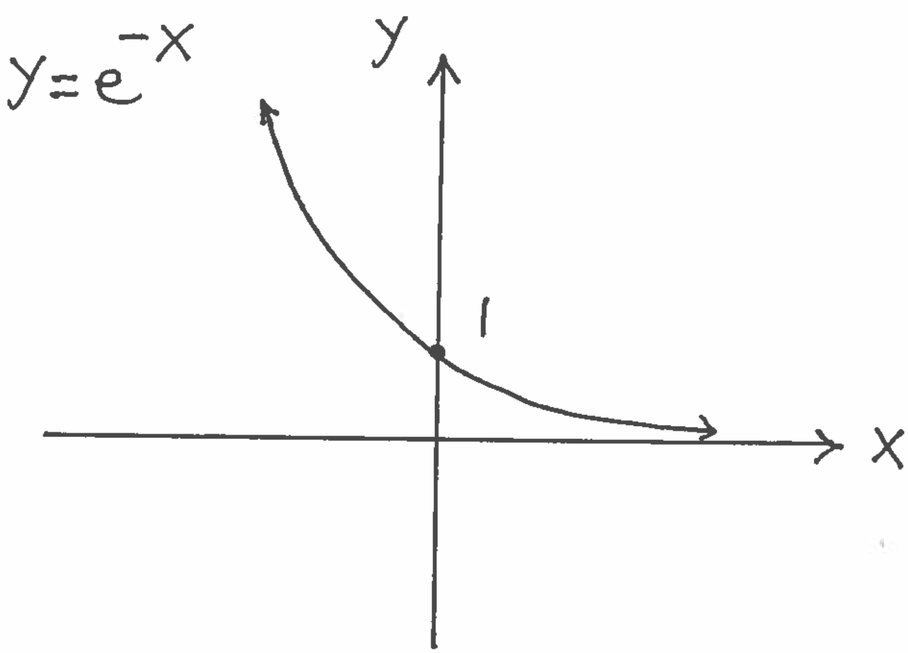
8.) $16^{1-t} = (8^t)^t$

ANS: $t = \frac{2}{3}$, $t = -2$

Graphs of $y = e^x$ and $y = e^{-x}$



NOTE: $e^x > 0$ for all x-values.



NOTE: $e^{-x} > 0$ for all x-values.