Example: Consider $\int_{-1}^{2} \frac{x}{x+2}\,dx$.

What can be said about the Absolute Simpson Error

$$|E_n| = \left| \int_{-1}^{2} \frac{x}{x+2}\,dx - S_n \right|$$

for a.) $n=5$?

b.) $n=20$?

Solution: $f(x) = \frac{x}{x+2} \xrightarrow{D}$

$$f'(x) = \frac{(x+2)(1) - x(1)}{(x+2)^2} = \frac{2}{(x+2)^2} = 2(x+2)^{-2}$$

$$\xrightarrow{D} f''(x) = -4(x+2)^{-3}$$

$$\xrightarrow{D} f'''(x) = 12(x+2)^{-4}$$

$$\xrightarrow{D} f^{(4)}(x) = -48(x+2)^{-5}$$

$$\longrightarrow \quad f^{(4)}(x) = \frac{-48}{(x+2)^5} \text{ ; then}$$

$$\max_{-1 \le x \le 2} |f^{(4)}(x)| = \max_{-1 \le x \le 2} \left| \frac{-48}{(x+2)^5} \right|$$

$$= \max_{-1 \le x \le 2} \frac{48}{|x+2|^5}$$

$$\le \frac{48}{|(-1)+2|^5} = 48 \text{ ; and}$$

$h = \dfrac{b-a}{n} = \dfrac{2-(-1)}{n} = \dfrac{3}{n}$; the absolute Simpson Error is

$$|E_n| \le (b-a)\frac{h^4}{180}\left\{\max_{a \le x \le b} |f^{(4)}(x)|\right\}$$

$$= (2-(-1)) \cdot \frac{\left(\frac{3}{n}\right)^4}{180} \cdot \{48\}$$

$$= \frac{3}{180} \cdot (81)\{48\} \cdot \frac{1}{n^4} \text{ , i.e.,}$$

$$|E_n| \le (64.8) \cdot \frac{1}{n^4}$$

a.) If $n=5$, then

$$|E_5| \le (64.8)\cdot\frac{1}{5^4} = 0.10368$$

b.) If $n=20$, then

$$|E_{20}| \le (64.8)\cdot\frac{1}{(20)^4} = 0.000405$$