Example: What should n be so that T_n, the Trapezoidal Estimate with n trapezoids, estimates the exact value of $\int_{-1}^{1} \ln(x+3)\,dx$ with an absolute Error of at most 0.001 ?

Solution: $f(x) = \ln(x+3) \xrightarrow{D}$

$f'(x) = \dfrac{1}{x+3} = (x+3)^{-1} \xrightarrow{D}$

$f''(x) = -(x+3)^{-2} = \dfrac{-1}{(x+3)^2}$; then

$$\max_{-1 \le x \le 1} |f''(x)| = \max_{-1 \le x \le 1} \left| \frac{-1}{(x+3)^2} \right|$$

$$\le \frac{1}{((-1)+3)^2} = \frac{1}{4} \text{ ; and}$$

$h = \frac{b-a}{n} = \frac{1-(-1)}{n} = \frac{2}{n}$; the absolute Trapezoidal Error is

$$|E_n| \le (b-a)\frac{h^2}{12}\left\{\max_{a \le x \le b} |f''(x)|\right\}$$

$$= (1-(-1))\,\frac{\left(\frac{2}{n}\right)^2}{12}\left\{\frac{1}{4}\right\}$$

$$= \frac{2}{12}\cdot\frac{4}{4}\cdot\frac{1}{n^2} = \frac{1}{6}\cdot\frac{1}{n^2}\text{, i.e.,}$$

$|E_n| \le \frac{1}{6}\cdot\frac{1}{n^2}$; now use the given error:

$$|E_n| \le \frac{1}{6}\cdot\frac{1}{n^2} \le 0.001 \longrightarrow$$

$$\frac{1}{6\cdot\frac{1}{1000}} \le n^2 \rightarrow n^2 \ge \frac{1000}{6} = \frac{500}{3} \longrightarrow$$

$n \ge \sqrt{\frac{500}{3}} \approx 12.9$ so choose

$$n = 13 .$$