The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–8, evaluate the indefinite integral.

1. \( \int 5 \, dx \)
2. \( \int \frac{1}{3} \, dx \)
3. \( \int x^{3/2} \, dx \)
4. \( \int x^{3/3} \, dx \)
5. \( \int 2x(x^2 + 1)^3 \, dx \)
6. \( \int 3x^2(x^3 - 1)^2 \, dx \)
7. \( \int 6e^x \, dx \)
8. \( \int \frac{2}{2x + 1} \, dx \)

In Exercises 9–12, simplify the expression.

9. \( 2x(x - 1)^2 + x(x - 1) \)
10. \( 6x(x + 4)^3 - 3x^2(x + 4)^2 \)
11. \( 3(x + 7)^{1/2} - 2x(x + 7)^{-1/2} \)
12. \( (x + 5)^{1/3} - 5(x + 5)^{-2/3} \)

In Exercises 13–38, find the indefinite integral.

1. \( \int (x - 2)^4 \, dx \)
2. \( \int (x + 5)^{3/2} \, dx \)
3. \( \int \frac{2}{(t - 9)^3} \, dt \)
4. \( \int \frac{4}{(1 - t)^5} \, dt \)
5. \( \int \frac{2t - 1}{t^2 - t + 2} \, dt \)
6. \( \int \frac{2y^3}{y^4 + 1} \, dy \)
7. \( \int \sqrt{1 + x} \, dx \)
8. \( \int (3 + x)^{3/2} \, dx \)
9. \( \int \frac{12x + 2}{3x^2 + x} \, dx \)
10. \( \int \frac{6x^2 + 2}{x^3 + x} \, dx \)
11. \( \int \frac{1}{(5x + 1)^3} \, dx \)
12. \( \int \frac{1}{(3x + 1)^2} \, dx \)
13. \( \int \frac{1}{\sqrt{x + 1}} \, dx \)
14. \( \int \frac{1}{\sqrt{5x + 1}} \, dx \)
15. \( \int \frac{e^{2x}}{1 - e^{3x}} \, dx \)
16. \( \int \frac{4e^{2x}}{1 + e^{2x}} \, dx \)
17. \( \int \frac{2x}{e^{2x}} \, dx \)
18. \( \int \frac{e^{\sqrt{x} + 1}}{\sqrt{x} + 1} \, dx \)
19. \( \int \frac{x^2}{x - 1} \, dx \)
20. \( \int \frac{2x}{x - 4} \, dx \)
21. \( \int x \sqrt{x^2 + 4} \, dx \)
22. \( \int \frac{t}{\sqrt{1 - t^2}} \, dt \)
23. \( \int e^{2x} \, dx \)
24. \( \int t e^{t^2} + 1 \, dt \)
25. \( \int \frac{e^{-x}}{e^{-x} + 2} \, dx \)
26. \( \int \frac{e^x}{1 + e^x} \, dx \)
27. \( \int \frac{x}{(x + 1)^4} \, dx \)
28. \( \int \frac{x^2}{(x + 1)^3} \, dx \)
29. \( \int \frac{x}{(3x - 1)^2} \, dx \)
30. \( \int \frac{5x}{(x - 4)^3} \, dx \)
31. \( \int \frac{1}{\sqrt{t} - 1} \, dt \)
32. \( \int \frac{1}{\sqrt{x + 1}} \, dx \)
33. \( \int \frac{2\sqrt{t} + 1}{t} \, dt \)
34. \( \int \frac{6x + \sqrt{x}}{x} \, dx \)
35. \( \int \frac{x}{\sqrt{2x + 1}} \, dx \)
36. \( \int \frac{x^2}{\sqrt{x - 1}} \, dx \)
37. \( \int t^2\sqrt{1 - t} \, dt \)
38. \( \int \sqrt{y + 1} \, dy \)

In Exercises 39–46, evaluate the definite integral.

39. \( \int_0^a \sqrt{2x + 1} \, dx \)
40. \( \int_0^b 4x + 1 \, dx \)
41. \( \int_0^1 3xe^{2x} \, dx \)
42. \( \int_0^2 e^{-2x} \, dx \)
43. \( \int_0^4 \frac{x}{(x + 4)^2} \, dx \)
44. \( \int_0^5 (x + 5)^4 \, dx \)
45. \( \int_0^{0.5} x(1 - x^2) \, dx \)
46. \( \int_0^{0.5} x^2(1 - x) \, dx \)

In Exercises 47–54, find the region bounded by the graphs of the equation and verify your answer.

47. \( y = x\sqrt{x - 3}, \quad y = \sqrt{2x + 1} \)
48. \( y = x\sqrt{2x + 1} \)
49. \( y = x^2\sqrt{1 - x} \)
50. \( y = x^2\sqrt{x + 2} \)
51. \( y = \frac{x^2 - 1}{\sqrt{x + 1}} \)
52. \( y = \frac{2x - 1}{\sqrt{x + 3}} \)
53. \( y = x\sqrt{x}, \quad y = \sqrt{x + 1} \)
54. \( y = x\sqrt{x + 2} \)

In Exercises 55–58, find the region bounded by the graphs of the equation.

55. \( y = -x\sqrt{x + 2} \)
56. \( y = x^3(1 - x^2) \)

(Hint: Find the area of the region bounded by \( y = x\sqrt{1 - x^2} \) and \( y = x\sqrt{x} \). Then multiply by 2.)

In Exercises 59 and 60, find the area of the region bounded by the curves.

59. \( y = x\sqrt{1 - x^2} \)
60. \( y = x\sqrt{1 - x^2} \)

In Exercises 61 and 62, find the function \( f(x) \) exceeds the function \( g(x) \).

61. \( f(x) = \frac{1}{x + 1}, \quad g(x) = \sqrt{4x + 1} \)
62. \( f(x) = x\sqrt{4x + 1} \)
In Exercises 47–54, find the area of the region bounded by the
graphs of the equations. Then use a graphing utility to graph the
region and verify your answer.

47. \( y = x\sqrt{x} - 3 \), \( y = 0 \), \( x = 7 \)
48. \( y = x^2 + 2x + 1 \), \( y = 0 \), \( x = 4 \)
49. \( y = x^2 \sqrt{1 - x} \), \( y = 0 \), \( x = -3 \)
50. \( y = x^2 \sqrt{x + 2} \), \( y = 0 \), \( x = 7 \)
51. \( y = \frac{x^2 - 1}{x} \), \( y = 0 \), \( x = 1 \), \( x = 5 \)
52. \( y = \frac{2x^2 - 1}{x^3} \), \( y = 0 \), \( x = \frac{1}{2} \), \( x = 6 \)
53. \( y = x^3 + 1 \), \( y = 0 \), \( x = 0 \), \( x = 2 \)
54. \( y = x^3 - 2x \), \( y = 0 \), \( x = 2 \), \( x = 10 \)

In Exercises 55–58, find the area of the region bounded by the
graphs of the equations.

55. \( y = -x\sqrt{x} + 2 \), \( y = 0 \)
56. \( y = x^3 - 1 \), \( y = 0 \)

57. \( y^2 = x^2(1 - x^2) \)
(Hint: Find the area of the region bounded by \( y = x\sqrt{1 - x^2} \) and \( y = 0 \). Then multiply by 4.)

58. \( y = 1/(1 + x) \), \( y = 0 \), \( x = 0 \), \( x = 4 \)

In Exercises 59 and 60, find the volume of the solid generated by
revolving the region bounded by the graph(s) of the equation(s)
about the \( x \)-axis.

59. \( y = x\sqrt{1 - x^2} \)
60. \( y = \sqrt{x}(1 - x^2) \), \( y = 0 \)

In Exercises 61 and 62, find the average amount by which the function
\( f \) exceeds the function \( g \) on the interval.

61. \( f(x) = \frac{1}{x + 1} \), \( g(x) = \frac{x}{(x + 1)^2} \), \([0, 1]\)
62. \( f(x) = x\sqrt{4x + 1} \), \( g(x) = 2\sqrt{x} \), \([0, 2]\)

**SECTION 6.1 Integration by Substitution**

### 63. Probability

The probability of recall in an experiment modeled by

\[
P(a \leq x \leq b) = \int_a^b \frac{15}{4} x \sqrt{1 - x} \, dx
\]

where \( x \) is the percent of recall (see figure).

(a) What is the probability of recalling between 40% and 80%?

(b) What is the median percent recall? That is, for which value of \( b \) is \( P(0 \leq x \leq b) = 0.5 \)?

![Figure for 63](image)

### 64. Probability

The probability of finding between \( a \) and \( b \) percent iron in ore samples is modeled by

\[
P(a \leq x \leq b) = \int_a^b \frac{1155}{32} x^3(1 - x)^{3/2} \, dx
\]

(see figure). Find the probabilities that a sample will contain between (a) 0% and 25% and (b) 50% and 100% iron.

### 65. Meteorology

During a two-week period in March in small town near Lake Erie, the measurable snowfall \( S \) (inches) on the ground can be modeled by

\[
S(t) = t\sqrt{14 - t}, \quad 0 \leq t \leq 14
\]

where \( t \) represents the day.

(a) Use a graphing utility to graph the function.

(b) Find the average amount of snow on the ground during the two-week period.

(c) Find the total snowfall over the two-week period.

### 66. Revenue

A company sells a seasonal product that generates a daily revenue \( R \) (in dollars per day) modeled by

\[
R = 0.06t^2(365 - t)^{1/2} + 1250, \quad 0 \leq t \leq 365
\]

where \( t \) represents the day.

(a) Find the average daily revenue over a period of 1 year.

(b) Describe a product whose seasonal sales pattern resembles the model. Explain your reasoning.

In Exercises 67 and 68, use a program similar to the Midpoint R program on page 366 with \( n = 10 \) to approximate the area of the region bounded by the graph(s) of the equation(s).

67. \( y = \sqrt{x} \sqrt{4 - x} \), \( y = 0 \)
68. \( y^2 = x^2(1 - x^2) \)
The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

**In Exercises 1–6, find \( f(x) \).**

1. \( f(x) = \ln(x+1) \)
2. \( f(x) = \ln(x^2 - 1) \)
3. \( f(x) = e^x \)
4. \( f(x) = e^{-x^2} \)
5. \( f(x) = x^2e^x \)
6. \( f(x) = xe^{-2x} \)

**In Exercises 7–10, find the area between the graphs of \( f \) and \( g \).**

7. \( f(x) = -x^2 + 4, \ g(x) = x^2 - 4 \)
8. \( f(x) = -x^2 + 2, \ g(x) = 1 \)
9. \( f(x) = 4x, \ g(x) = x^2 - 5 \)
10. \( f(x) = x^3 - 3x^2 + 2, \ g(x) = x - 1 \)

**Exercises 6.2**

In Exercises 1–6, use integration by parts to find the indefinite integral.

1. \( \int xe^x \, dx \)
2. \( \int xe^{-x} \, dx \)
3. \( \int x^2e^{-x} \, dx \)
4. \( \int xe^{2x} \, dx \)
5. \( \int \ln 2x \, dx \)
6. \( \int \ln x^2 \, dx \)

In Exercises 7–28, find the indefinite integral. (Hint: Integration by parts is not required for all the integrals.)

7. \( \int e^{4x} \, dx \)
8. \( \int e^{-2x} \, dx \)
9. \( \int xe^{4x} \, dx \)
10. \( \int xe^{-2x} \, dx \)
11. \( \int xe^{x^2} \, dx \)
12. \( \int x^2e^{x^2} \, dx \)
13. \( \int x^2e^x \, dx \)
14. \( \int \frac{e^x}{x} \, dx \)
15. \( \int t \ln(t + 1) \, dt \)
16. \( \int x^3 \ln x \, dx \)
17. \( \int \frac{e^{1/t}}{t^2} \, dt \)
18. \( \int \frac{1}{x(\ln x)^3} \, dx \)
19. \( \int x(\ln x)^2 \, dx \)
20. \( \int \ln 3x \, dx \)
21. \( \int \frac{(\ln x)^2}{x} \, dx \)
22. \( \int \frac{1}{x \ln x} \, dx \)
23. \( \int x\sqrt{x-1} \, dx \)
24. \( \int \frac{x}{\sqrt{x-1}} \, dx \)
25. \( \int x(x+1)^2 \, dx \)
26. \( \int \frac{x}{\sqrt{2 + 3x}} \, dx \)
27. \( \int \frac{x e^{2x}}{(2x+1)^2} \, dx \)
28. \( \int \frac{x^2e^{x^2}}{(x^2 + 1)^2} \, dx \)

In Exercises 29–34, evaluate the definite integral.

29. \( \int_{0}^{1} x^2e^x \, dx \)
30. \( \int_{0}^{2} x^2 \, dx \)
31. \( \int_{1}^{2} x^3 \ln x \, dx \)
32. \( \int_{0}^{1} 2x \ln x \, dx \)
33. \( \int_{-1}^{0} \ln(x + 2) \, dx \)
34. \( \int_{0}^{1} \ln(1 + 2x) \, dx \)

In Exercises 35–38, find the area of the region bounded by the graphs of the equations. Then use a graphing utility to graph the region and verify your answer.

35. \( y = x^3e^x, \ y = 0, \ x = 0, \ x = 2 \)
36. \( y = (x^2 - 1)e^x, \ y = 0, \ x = -1, \ x = 1 \)
37. \( y = x^2 \ln x, \ y = 0, \ x = 1, \ x = e \)
38. \( y = \frac{\ln x}{x^2}, \ y = 0, \ x = 1, \ x = e \)
In Exercises 39–42, find the indefinite integral using each specified method. Then write a brief statement explaining which method you prefer.

39. \[ \int 2x\sqrt{2x - 3} \, dx \]
   (a) By parts, letting \( dv = \sqrt{2x - 3} \, dx \)
   (b) By substitution, letting \( u = \sqrt{2x - 3} \)

40. \[ \int x\sqrt{4 + x} \, dx \]
   (a) By parts, letting \( dv = \sqrt{4 + x} \, dx \)
   (b) By substitution, letting \( u = \sqrt{4 + x} \)

41. \[ \int \frac{x}{\sqrt{4 + 5x}} \, dx \]
   (a) By parts, letting \( dv = \frac{1}{\sqrt{4 + 5x}} \, dx \)
   (b) By substitution, letting \( u = \sqrt{4 + 5x} \)

42. \[ \int x\sqrt{4 - x} \, dx \]
   (a) By parts, letting \( dv = \sqrt{4 - x} \, dx \)
   (b) By substitution, letting \( u = \sqrt{4 - x} \)

In Exercises 43 and 44, use integration by parts to verify the formula.

43. \[ \int x^n \ln x \, dx = \frac{x^{n+1}}{n+1} \left[-1 + (n+1) \ln x\right] + C, \quad n \neq -1 \]

44. \[ \int x^n e^{ax} \, dx = \frac{x^{n+1} e^{ax}}{a} - \frac{n}{a} \int x^{n-1} e^{ax} \, dx \]

In Exercises 45–48, use the results of Exercises 43 and 44 to find the indefinite integral.

45. \[ \int x^2 e^{3x} \, dx \]
46. \[ \int xe^{-3x} \, dx \]
47. \[ \int x^2 \ln x \, dx \]
48. \[ \int x^{1/2} \ln x \, dx \]

In Exercises 49–52, find the area of the region bounded by the graphs of the given equations.

49. \( y = xe^{-x}, \quad y = 0, \quad x = 4 \)
50. \( y = \frac{1}{2}xe^{-x/3}, \quad y = 0, \quad x = 0, \quad x = 3 \)
51. \( y = x \ln x, \quad y = 0, \quad x = e \)
52. \( y = x^3 \ln x, \quad y = 0, \quad x = e \)

53. Given the region bounded by the graphs of \( y = 2 \ln x, \quad y = 0, \quad \) and \( x = e \) (see figure), find
   (a) the area of the region.
   (b) the volume of the solid generated by revolving the region about the \( x \)-axis.

54. Given the region bounded by the graphs of \( y = xe^t, \quad y = 0, \quad x = 0, \quad x = 1 \) (see figure), find
   (a) the area of the region.
   (b) the volume of the solid generated by revolving the region about the \( x \)-axis.

55. \[ \int_0^t r^3 e^{-rt} \, dt \]
56. \[ \int_0^1 \ln(x^2 + 4) \, dx \]
57. \[ \int_0^5 x^4(5 - x^2)^{1/2} \, dx \]
58. \[ \int_0^1 x^2 \ln x \, dx \]

59. **Demand** A manufacturing company forecasts that the demand \( x \) (in units per year) for its product over the next 10 years can be modeled by \( x = 500(20 + te^{-0.1t}) \) for \( 0 \leq t \leq 10 \), where \( t \) is the time in years.
   (a) Use a graphing utility to decide whether the company is forecasting an increase or a decrease in demand over the decade.
   (b) According to the model, what is the total demand over the next 10 years?
   (c) Find the average annual demand during the 10-year period.

60. **Capital Campaign** The board of trustees of a college is planning a five-year capital gifts campaign to raise money for the college. The goal is to have an annual gift income \( I \) that is modeled by \( I = 2000(375 + 68te^{-0.2t}) \) for \( 0 \leq t \leq 5 \), where \( t \) is the time in years.
(a) Use a graphing utility to decide whether the board of trustees expects the gift income to increase or decrease over the five-year period.
(b) Find the expected total gift income over the five-year period.
(c) Determine the average annual gift income over the five-year period. Compare the result with the income given when \( t = 3 \).

61. Learning Theory A model for the ability \( M \) of a child to memorize, measured on a scale from 0 to 10, is
\[
M = 1 + 1.6t \ln t, \quad 0 < t \leq 4
\]
where \( t \) is the child’s age in years. Find the average value of this model between
(a) the child’s first and second birthdays.
(b) the child’s third and fourth birthdays.

62. Revenue A company sells a seasonal product. The revenue \( R \) (in dollars per year) generated by sales of the product can be modeled by
\[
R = 410.5t^2e^{-t/50} + 25,000, \quad 0 \leq t \leq 365
\]
where \( t \) is the time in days.
(a) Find the average daily receipts during the first quarter, which is given by \( 0 \leq t \leq 90 \).
(b) Find the average daily receipts during the fourth quarter, which is given by \( 274 \leq t \leq 365 \).
(c) Find the total daily receipts during the year.

70. Present Value A professional athlete signs a three-year contract in which the earnings can be modeled by
\[
c = 300,000 + 125,000t.
\]
(a) Find the actual value of the athlete’s contract.
(b) Assuming an annual inflation rate of 5%, what is the present value of the contract?

Future Value In Exercises 71 and 72, find the future value of the income in dollars given by \( f(t) \) over \( t \) years at the annual interest rate of \( r \).
If the function \( f \) represents a continuous investment over a period of \( t \) years at an annual interest rate of \( r \) (compounded continuously), then the future value of the investment is given by
\[
\text{Future value} = e^{rt} \int_0^t f(t)e^{-rt} \, dt.
\]
71. \( f(t) = 3000, \quad r = 8\%, \quad t = 10 \) years
72. \( f(t) = 3000e^{0.05t}, \quad r = 10\%, \quad t = 5 \) years

73. Finance: Future Value Use the equation from Exercises 71 and 72 to calculate the following. (Source: Adapted from Garman/Forgue, Personal Finance, Fifth Edition)
(a) The future value of $1200 saved each year for 10 years earning 7% interest.
(b) A person who wishes to invest $1200 each year finds one investment choice that is expected to pay 9% interest per year and another, riskier choice that may pay 10% interest per year. What is the difference in return (future value) if the investment is made for 15 years?

74. Consumer Awareness In 2004, the total cost to attend Pennsylvania State University for 1 year was estimated to be $19,843. If your grandparents had continuously invested in a college fund according to the model
\[
f(t) = 400t
\]
for 18 years, at an annual interest rate of 10%, would the fund have grown enough to allow you to cover 4 years of expenses at Pennsylvania State University? (Source: Pennsylvania State University)

75. Use a program similar to the Midpoint Rule program on page 366 with \( n = 10 \) to approximate
\[
\int_0^4 \frac{4}{\sqrt{x + \sqrt{x}} \, dx}.
\]
76. Use a program similar to the Midpoint Rule program on page 366 with \( n = 12 \) to approximate the volume of the solid generated by revolving the region bounded by the graphs of
\[
y = \frac{10}{\sqrt{x}}, \quad y = 0, \quad x = 1, \quad \text{and} \quad x = 4
\]
about the \( x \)-axis.