Measuring the Intensity of Sound

The **intensity level** of sound measured in decibels (dB)* is given by

$$b = 10 \log_{10} \frac{I}{I_0}. \tag{2}$$

Here I_0 is a reference intensity of 10^{-16} watt/cm^2 corresponding to approximately the faintest sound that can be heard. When $I = I_0$ then (2) gives $b = 0$ dB. The intensity levels of frequently occurring sounds are given in the table on page 176.

*The decibel is (1/10) bell. This latter unit, named for Alexander Graham Bell (1847–1922), proved to be too large in practice.

Source: College Mathematics by Dennis Zill, 2nd Edition

Source	Intensity level (dB)
Threshold of hearing	0
Whisper	20
Normal talking	40–60
Some TV commercials	65
Smoke detector alarm	70
Jet airplane taking off	80–100
Threshold of pain	120

Example Determine the intensity level of a sound having intensity 10^{-4} watt/cm^2.

Solution From (2) we see that the intensity level is

$$b = 10 \log_{10} \frac{10^{-4}}{10^{-16}} = 10 \log_{10} 10^{12}$$
$$= 120 \text{ dB}.$$

As indicated in the table, sound at an intensity level of around 120 dB can cause pain. Prolonged exposure to levels around 90 dB (easily produced by rock and roll groups) can cause temporary deafness.

Equation (2) can be used to obtain the intensity level b_2 of a sound at a distance d_2 from the source if one level b_1 is measured at a distance d_1. Now it is known that the intensity of sound I is inversely proportional to the square of the distance d from the source:

$$I = \frac{k}{d^2} \tag{3}$$

where k is the intensity at a unit distance from the source. Substituting (3) into (2) and using properties II and III of logarithms gives

$$b = 10 \log_{10} \frac{k/d^2}{I_0} = \log_{10} \frac{k/I_0}{d^2}$$
$$= 10 \,[\log_{10} k/I_0 - \log_{10} d^2]$$
$$= 10 \,[\log_{10} k/I_0 - 2 \log_{10} d].$$

Hence at d_2 and d_1,

$$b_2 = 10 \,[\log_{10} k/I_0 - 2 \log_{10} d_2] \tag{4}$$

$$b_1 = 10 \,[\log_{10} k/I_0 - 2 \log_{10} d_1]. \tag{5}$$

Subtracting (5) from (4) then yields

$$b_2 - b_1 = 10\ [2 \log_{10} d_1 - 2 \log_{10} d_2]$$
$$= 20 \log_{10} d_1/d_2$$

or

$$b_2 = b_1 + 20 \log_{10} \frac{d_1}{d_2}. \qquad (6)$$

AIRPORT NOISE REDUCTION In an experimental two-segment approach to an airport a plane starts a 6-degree glide path at 5.5 miles out from the runway and switches to a 3-degree glide path 1.5 miles out. A normal approach consists of a 3-degree glide path starting 5.5 miles out from the runway. The obvious purpose of the two-segment approach is that one plane causes less noise simply because it is higher. It is readily shown that at the 5.5-mile point the higher plane P_2, shown in Figure 5.5, part a, is at an altitude of 2635 feet whereas P_1, shown in Figure 5.5, part b, at the same point, has an altitude of 1522 feet. Equation (3) then implies that at this starting point of both glide paths, the sound intensity I of P_2 is one-third that of P_1. However, this does *not* mean that the *intensity level* of P_2 is one-third that of the intensity level of P_1.

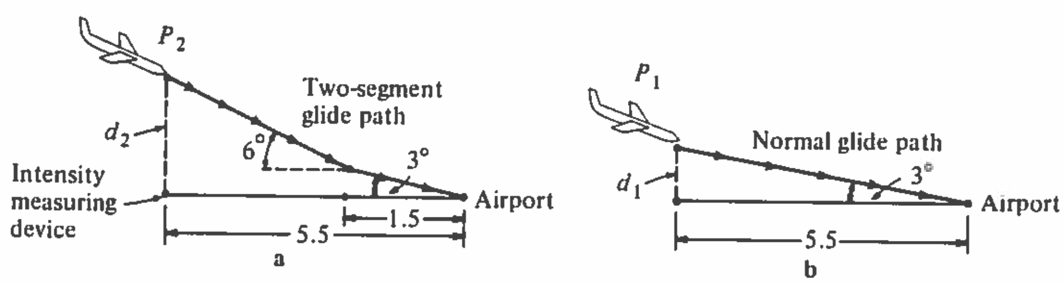

Figure 5.5

Example The intensity level b_1 of plane P_1 5.5 miles out from the runway is measured on ground level at 80 dB. Determine the intensity level b_2 of plane P_2 at the same point.

Solution Identifying $d_1 = 1522$ and $d_2 = 2635$, it follows from (6) that

$$b_2 = 80 + 20 \log_{10} \frac{1522}{2635}$$
$$= 80 + 20 \log_{10} 0.58$$
$$= 80 - 4.7314$$
$$\approx 75 \text{ dB}.$$