1. PLEASE DO NOT TURN THIS PAGE UNTIL TOLD TO DO SO.

2. IT IS A VIOLATION OF THE UNIVERSITY HONOR CODE TO, IN ANY WAY, ASSIST ANOTHER PERSON IN THE COMPLETION OF THIS EXAM. IT IS A VIOLATION OF THE UNIVERSITY HONOR CODE TO TAKE AN EXAM FOR ANOTHER PERSON. PLEASE KEEP YOUR OWN WORK COVERED UP AS MUCH AS POSSIBLE DURING THE EXAM SO THAT OTHERS WILL NOT BE TEMPTED OR DISTRACTED. THANK YOU FOR YOUR COOPERATION.

3. No notes, books, or classmates may be used as resources for this exam. YOU MAY USE A CALCULATOR ON THIS EXAM.

4. Read directions to each problem carefully. Show all work for full credit. In most cases, a correct answer with no supporting work will receive little or no credit. What you write down and how you write it are the most important means of your getting a good score on this exam. Neatness and organization are also important.

5. Make sure that you have 7 pages, including the cover page.

6. Include units on answers where units are appropriate.

7. You have until 8:50 a.m. sharp to finish the exam.

8. Please stop immediately and close your exam when I call time at the end. Failure to stop working and closing your exam in a timely fashion may lead to points being deducted from your exam score. Thank you for your cooperation.
1.) (2 pts. each) Determine whether each statement is true (T) or false (F). Then circle the appropriate response.

a.) \( \frac{1}{x+y} = \frac{1}{x} + \frac{1}{y} \)  
\[ \text{T } \bigcirc \ F \]

b.) \((\ln x)^m = \ln(mx)\)  
\[ \text{T } \bigcirc \ F \]

c.) \(\sqrt{x+y} = \sqrt{x} + \sqrt{y}\)  
\[ \text{T } \bigcirc \ F \]

d.) \(\frac{\ln x}{\ln y} = \ln x - \ln y\)  
\[ \text{T } \bigcirc \ F \]

2.) (7 pts.) Solve the following equation for \( t \) : \( e^{2t} = 3e^t + 4 \)

\[
e^{2t} - 3e^t - 4 = 0 \rightarrow (e^t)^2 - 3(e^t) - 4 = 0
\]
\[
\rightarrow (e^t - 4)(e^t + 1) = 0
\]
\[
\rightarrow e^t = 4 \rightarrow \boxed{t = \ln 4} \quad \text{or}
\]
\[
e^t = -1 \quad (\text{NO!})
\]

3.) Let \( y = xe^{-x} \).

a.) (4 pts.) Solve \( y' = 0 \) for \( x \).

\[ \frac{D}{Dx} y' = \frac{D}{Dx} (x \cdot (-e^{-x}) + (1) \cdot e^{-x}) = e^{-x} (1-x) = 0 \]
\[
\rightarrow e^{-x} = 0 \quad (\text{NO!}) \quad \text{or} \quad \boxed{x = 1}
\]

b.) (4 pts.) Solve \( y'' = 0 \) for \( x \).

\[ \frac{D}{Dx} y'' = e^{-x} (-1) + (-e^{-x}) (1-x)
\]
\[
= -e^{-x} (1+1-x) = -e^{-x} (2-x) = 0
\]
\[
\rightarrow e^{-x} = 0 \quad (\text{NO!}) \quad \text{or} \quad \boxed{x = 2}
\]
4.) A bowling ball is dropped (Initial velocity is 0 ft./sec.) from a tall building and it strikes the ground in 5 seconds. Assume that the acceleration due to gravity is $-32$ ft./sec.$^2$  

Let $s(t)$ = height (ft.) at time $t$ (sec.)

a.) (4 pts.) Derive formulas for the velocity and height (above the ground) of the doomed bowling ball. 

$$s''(t) = -32 \rightarrow s'(t) = -32t + c \text{ and}$$

$$s'(0) = 0 \rightarrow 0 = -32(0) + c \rightarrow c = 0 \text{ so vel. is}$$

$$s'(t) = -32t \rightarrow s(t) = -16t^2 + c \text{ and}$$

$$s(5) = 0 \rightarrow 0 = -16(5)^2 + c \rightarrow c = 400 \rightarrow s(t) = -16t^2 + 400$$

b.) (2 pts.) How tall is the building ?

**Building Height:** $s(0) = -16(0)^2 + 400$

$= 400$ ft.

c.) (2 pts.) What is the bowling ball’s velocity as it strikes the ground ?

**Velocity:** $s'(5) = -32(5)$

$= -160$ ft./sec.

5.) (8 pts.) Cobalt-60 is a metallic isotope used to sterilize medical supplies and medical waste. It’s half-life is about 5.27 years. If a sample of Co-60 has a mass of 15 mg. after 20 years, what was the initial amount of Co-60 ?

$$A = Ce^{kt} \text{ and } t = 5.27, A = \frac{1}{2}C \text{ then}$$

$$\frac{1}{2}C = C e^{5.27k} \rightarrow \ln \frac{1}{2} = \ln e^{5.27k} = 5.27k \rightarrow$$

$$k = \frac{\ln \frac{1}{2}}{5.27} j \text{ and } t = 20, A = 15 \text{ then}$$

$$15 = C e^{\frac{\ln \frac{1}{2}}{5.27}(20)} \rightarrow C = \frac{15}{e^{\frac{20\ln \frac{1}{2}}{5.27}}} \approx 208.22 \text{ mg}$$
6.) (7 pts.) Let \( f(x) = x \ln x \). Set up a sign chart for the first derivative, \( f'(x) \), to determine any relative maximum or minimum points, \((x, y)\), for the graph of \( f \).

\[
D \rightarrow f'(x) = x \cdot \frac{1}{x} + (1) \ln x
\]
\[
= 1 + \ln x = 0 \rightarrow \ln x = -1 \rightarrow x = e^{-1}
\]

\[
\begin{array}{c|c|c|c}
& - & 0 & + \\
x = 0 & x = e^{-1} & 1 & e \\
\end{array}
\]
\[
\begin{align*}
A(x) &= e^{-1} \ln e^{-1} = -e^{-1} \\
\text{MIN}
\end{align*}
\]

7.) (8 pts.) You invest $1000 in an account earning an annual interest rate of 8.5%. If interest is compounded monthly, how many years will it take for this amount to grow to $2500?

\[
A = P \left(1 + \frac{r}{n}\right)^{nt} \rightarrow 2500 = 1000 \left(1 + \frac{0.085}{12}\right)^{12t}
\]
\[

t = \frac{\ln 2.5}{12 \ln \left(1 + \frac{0.085}{12}\right)} \approx 10.818 \text{ yrs.}
\]
8.) (8 pts.) Use implicit differentiation to determine \( y' = \frac{dy}{dx} \) for \( e^{x+y} + 3^y = 2x - 5y + 6 \).

\[
D \quad e^{x+y} (1 + y') + 3^y y' \ln 3 = 2 - 5y' + 0
\]

\[
\rightarrow e^{x+y} y' + 3^y y' \ln 3 = 2 - 5y'
\]

\[
\rightarrow e^{x+y} y' + 3^y y' \ln 3 + 5y' = 2 - e^{x+y}
\]

\[
\rightarrow y' \left[ e^{x+y} + 3^y \ln 3 + 5 \right] = 2 - e^{x+y}
\]

\[
\rightarrow y' = \frac{2 - e^{x+y}}{e^{x+y} + 3^y \ln 3 + 5}
\]

9.) (6 pts.) Determine if the following antiderivative is true (T) or false (F). Give supporting steps or reasons for your answer.

\[
\int 2x(x \sec^2 2x + \tan 2x) \, dx = x^2 \tan 2x + C
\]

\[
D \quad (x^2 \tan 2x + C)
\]

\[
= x^2 \sec^2 2x \cdot 2 + (2x) \tan 2x
\]

\[
= 2x (x \sec^2 2x + \tan 2x)
\]

so \( \text{TRUE} \)
10.) (8 pts. each) Determine the following antiderivatives.

a.) \[ \int \frac{x^4 + 7}{x^3} \, dx = \int \left( \frac{x^4}{x^3} + \frac{7}{x^3} \right) \, dx \]
\[ = \int (x + 7x^{-3}) \, dx \]
\[ = \frac{1}{2} x^2 + 7 \cdot x^{-2} + C \]

b.) \[ \int (x + 1)(x^2 + 2x)^{10} \, dx \]
\[ \text{(Let } u = x^2 + 2x \quad \rightarrow \quad du = (2x + 2) \, dx = 2(x + 1) \, dx) \]
\[ = \frac{1}{2} \int u^{10} \, du \]
\[ = \frac{1}{2} \cdot \frac{1}{11} u^{11} + C = \frac{1}{22} (x^2 + 2x)^{11} + C \]

c.) \[ \int \frac{1}{\sqrt{x} (\sqrt{x} + 9)^2} \, dx \]
\[ \text{(Let } u = \sqrt{x} + 9 \quad \rightarrow \quad du = \frac{1}{2} \sqrt{x} \, dx \rightarrow 2 \, du = \frac{1}{\sqrt{x}} \, dx) \]
\[ = 2 \int \frac{1}{u^2} \, du = 2 \int u^{-2} \, du = 2 \cdot u^{-1} + C \]
\[ = -2 (\sqrt{x} + 9) + C \]

d.) \[ \int (x^2 - x)(x + 1) \, dx = \int (x^3 + x^2 - x^2 - x) \, dx \]
\[ = \int (x^3 - x) \, dx = \frac{1}{4} x^4 - \frac{1}{2} x^2 + C \]
EXTRA CREDIT PROBLEM– The following problem is worth 8 points. This problem is OPTIONAL.

1.) Determine the following indefinite integral.

\[
\int \frac{1}{\sqrt{x+1} - \sqrt{x}} \, dx
\]

\[
= \int \frac{1}{\sqrt{x+1} - \sqrt{x}} \cdot \frac{\sqrt{x+1} + \sqrt{x}}{\sqrt{x+1} + \sqrt{x}} \, dx
\]

\[
= \int \frac{\sqrt{x+1} + \sqrt{x}}{(x+1) - x} \, dx
\]

\[
= \int \left( (x+1)^{1/2} + x^{1/2} \right) \, dx
\]

\[
= \frac{2}{3} (x+1)^{3/2} + \frac{2}{3} x^{3/2} + C
\]