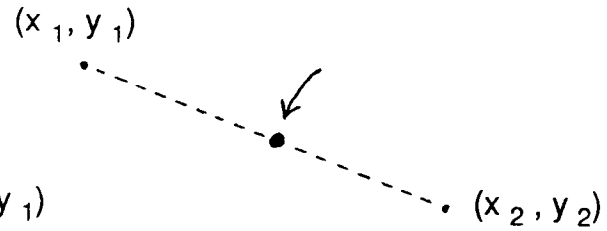


Math 16C
 Kouba
 Three-Dimensional (3-D) Space

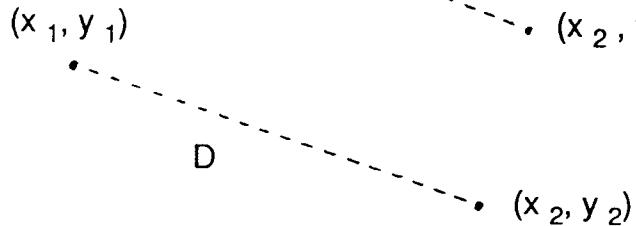
RECALL : Consider two points (x_1, y_1) and (x_2, y_2) in two-dimensional space. The midpoint of the line segment joining these two points is given by

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right).$$



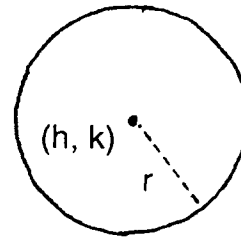
The distance between these two points is

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$



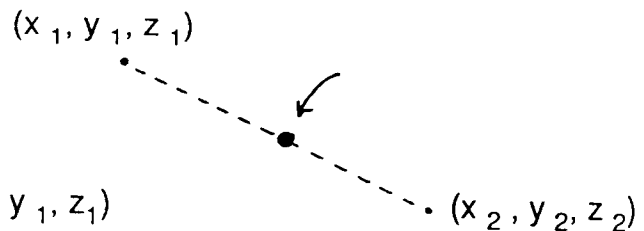
RECALL : The set of all points (x, y) in two-dimensional space which are a distance r from a fixed point (h, k) is a circle (with center (h, k) and radius r) given by the equation

$$(x - h)^2 + (y - k)^2 = r^2.$$



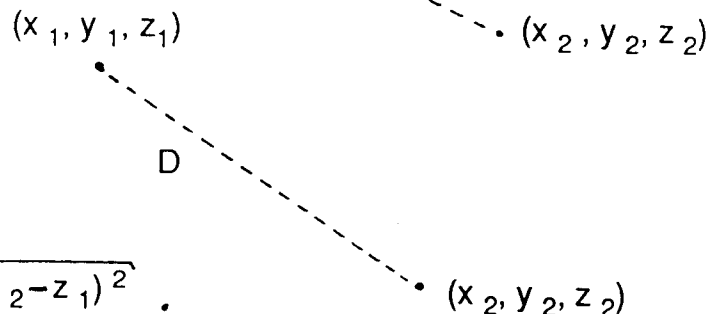
Let (x_1, y_1, z_1) and (x_2, y_2, z_2) be two points in three-dimensional space. The midpoint of the line segment joining these two points is given by

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right).$$



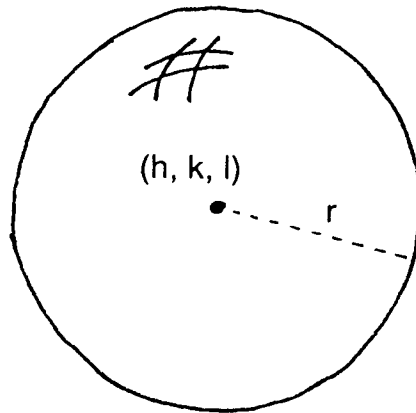
The distance between these two points is

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.$$



DEFINITION : The set of all points (x, y, z) in three-dimensional space which are a distance r from a fixed point (h, k, l) is a sphere (with center (h, k, l) and radius r) given by the equation

$$(x-h)^2 + (y-k)^2 + (z-l)^2 = r^2.$$



Example : Find the center and radius of each of the following spheres.

1. $2x^2 + 2y^2 + 2z^2 = 32$

center $(0, 0, 0)$

radius 4

2. $x^2 + y^2 + z^2 - 4x + 6y = 17$

center $(2, -3, 0)$

radius $\sqrt{30}$

Example : The diameter of a sphere has endpoints $(1, 3, 0)$ and $(-2, 4, 6)$. Determine an equation for this sphere.

$$(x + 1/2)^2 + (y - 7/2)^2 + (z - 3)^2 = 23/2$$

Example : Find and simplify an equation for all points (x, y, z) in three-dimensional space which are equidistant from the point $(1, -2, 3)$ and the plane $z = -1$.

$$z = 1/8 (x - 1)^2 + 1/8 (y + 2)^2 + 1$$