Math 16C  
Kouba  
Three-Dimensional (3-D) Space

**RECALL**: Consider two points \((x_1, y_1)\) and \((x_2, y_2)\) in two-dimensional space. The midpoint of the line segment joining these two points is given by

\[
\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right).
\]

The distance between these two points is

\[
D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}. 
\]

**RECALL**: The set of all points \((x, y)\) in two-dimensional space which are a distance \(r\) from a fixed point \((h, k)\) is a circle (with center \((h, k)\) and radius \(r\)) given by the equation

\[
(x - h)^2 + (y - k)^2 = r^2.
\]

Let \((x_1, y_1, z_1)\) and \((x_2, y_2, z_2)\) be two points in three-dimensional space. The midpoint of the line segment joining these two points is given by

\[
\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right).
\]

The distance between these two points is

\[
D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.
\]
**DEFINITION**: The set of all points \((x, y, z)\) in three-dimensional space which are a distance \(r\) from a fixed point \((h, k, l)\) is a sphere (with center \((h, k, l)\) and radius \(r\)) given by the equation

\[
(x - h)^2 + (y - k)^2 + (z - l)^2 = r^2.
\]

**Example**: Find the center and radius of each of the following spheres.

1. \(2x^2 + 2y^2 + 2z^2 = 32\)  
   - center \((0, 0, 0)\)  
   - radius 4

2. \(x^2 + y^2 + z^2 - 4x + 6y = 17\)  
   - center \((2, -3, 0)\)  
   - radius \(\sqrt{30}\)

**Example**: The diameter of a sphere has endpoints \((1, 3, 0)\) and \((-2, 4, 6)\). Determine an equation for this sphere.

\[
(x + 1/2)^2 + (y - 7/2)^2 + (z - 3)^2 = 23/2
\]

**Example**: Find and simplify an equation for all points \((x, y, z)\) in three-dimensional space which are equidistant from the point \((1, -2, 3)\) and the plane \(z = -1\).

\[
z = 1/8 (x - 1)^2 + 1/8 (y + 2)^2 + 1
\]