Example : Assume that the number A of insects in a small population at time t (weeks) changes at a rate proportional to the square of the number present. If initially there are 250 insects and after 3 weeks there are 420, how many insects do you expect after 7 weeks ?

$$\frac{dA}{dt} = kA^2 \rightarrow \int \frac{1}{A^2}\, dA = \int k\, dt \rightarrow$$

$$\frac{-1}{A} = kt + c \rightarrow A = \frac{-1}{kt+c} ;$$

if $t=0$, $A=250$ then $250 = \frac{-1}{c} \rightarrow c = \frac{-1}{250} \rightarrow$

$$A = \frac{-1}{kt - \frac{1}{250}} = \frac{250}{-250kt+1} ;$$

if $t=3$, $A=420$ then $420 = \frac{250}{-750k+1} \rightarrow$

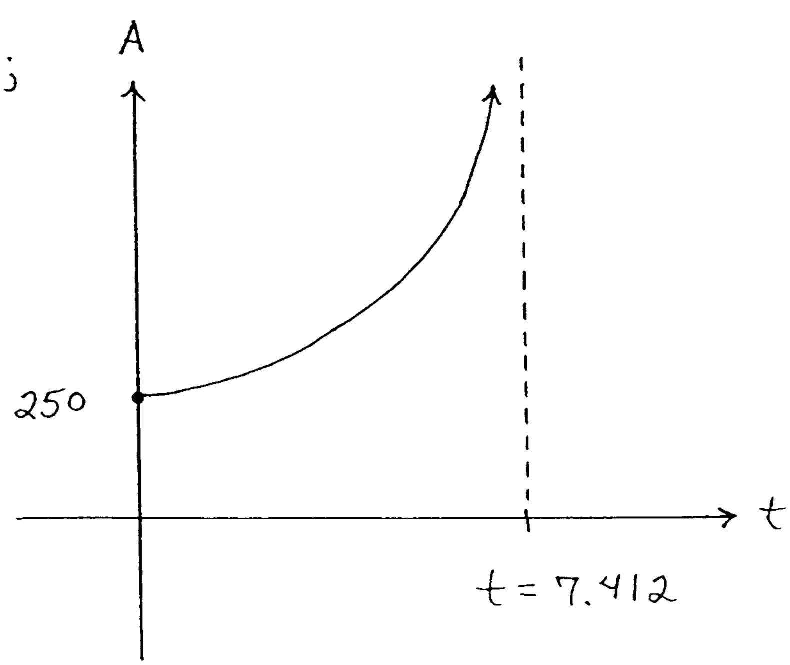
$$k = \frac{1}{750}\left(1 - \frac{250}{420}\right) = \frac{17}{31{,}500} = 0.000539682 \rightarrow$$

$$A = \frac{250}{1 - 0.134920634\,t} ;$$

if $t = 7$ weeks

then $A \approx 4500$

insects .



Example: Assume that the number P of elk at time t (years) on a large game preserve changes at a rate proportional to the product of P and 500 − P. Initially there are 85 elk and after 5 years there are 130. How many elk do you expect after 10 years?

$$\frac{dA}{dt} = kP(500-P) \rightarrow \int \frac{1}{P(500-P)}\,dP = \int k\,dt \rightarrow$$

$$\int \left[\frac{\frac{1}{500}}{P} + \frac{\frac{1}{500}}{500-P} \right] dP = kt + c \rightarrow$$

$$\frac{1}{500}\ln P - \frac{1}{500}\ln(500-P) = kt + c \rightarrow$$

$$\ln P - \ln(500-P) = 500kt + 500c \rightarrow$$

$$\ln \frac{P}{500-P} = 500kt + c \rightarrow$$

$$\frac{P}{500-P} = e^{c+500kt} = e^{c}e^{500kt} = ce^{500kt} \rightarrow$$

$$\frac{P}{500-P} = ce^{500kt}; \quad \text{if } t=0,\ P=85 \text{ then}$$

$$\frac{85}{415} = c = 0.204819277 \rightarrow \frac{P}{500-P} = 0.204819277 \cdot e^{500kt};$$

$$\text{if } t=5,\ P=130 \text{ then} \quad \frac{130}{370} = 0.204819277 \cdot e^{2500k} \rightarrow$$

$$k = 0.000215863 \quad (\text{why ?}) \rightarrow$$

$$\frac{P}{500-P} = 0.204819277 \cdot e^{0.1079315t};$$

Solve explicitly for P. First,

$$P = 102.4096385\, e^{0.1079315t} - 0.204819277 \cdot e^{0.1079315t} \cdot P \rightarrow$$

$$\left(1 + 0.204819277 \cdot e^{0.1079315t}\right) P = 102.4096385 \cdot e^{0.1079315t} \rightarrow$$

$$P = \frac{102.4096385 \cdot e^{0.1079315t}}{1 + 0.204819277 \cdot e^{0.1079315t}} \rightarrow$$

$$P = \frac{102.4096385}{e^{-0.1079315t} + 0.204819277} \quad ;$$

if $t = 10$ years then $P \approx 188$ elk.

Note that $\lim\limits_{t \to +\infty} P = 500$ (why?).