Math 16C
Kouba
Optional Practice Problems– Applications of D.E.'s

EXAMPLE 1: Assume that the outside air temperature T (^oF) at Lake Tahoe is a function of time t ($hrs.$), and that the rate of change of T is directly proportional to the time t. In addition, if $t = 1$ $hr.$, then $T = 25$ ^oF and if $t = 3$ $hrs.$, then $T = 41$ ^oF.

a.) Write an equation for the Differential Equation.
b.) Solve the Differential Equation.
c.) Write T explicitly as a function of t.
d.) What is T if $t = 4$ $hrs.$?

EXAMPLE 2: Assume that the volume V ($in.^3$) of a weather balloon is a function of pressure x ($lbs./in.^2$), and that the rate of change of V is inversely proportional to the volume V. In addition, if $x = 10$ $lbs./in.^2$, then $V = 40$ $in.^3$, and if $x = 15$ $lbs./in.^2$, then $V = 30$ $in.^3$

a.) Write an equation for the Differential Equation.
b.) Solve the Differential Equation.
c.) Write V explicitly as a function of x.
d.) What is V if $x = 18$ $lbs./in.^2$?

Example 1:

a.) $\frac{dT}{dt} = kt$

b.) $\int dT = \int kt\, dt \longrightarrow$

$T = k \cdot \frac{1}{2} t^2 + C$

c.) $\begin{cases} t=1, & T=25 \\ t=3, & T=41 \end{cases} \longrightarrow \begin{cases} 25 = \frac{1}{2}k + C \\ 41 = \frac{9}{2}k + C \end{cases}$

$\longrightarrow C = 25 - \frac{1}{2}k \longrightarrow$ (SUB) $\longrightarrow$

$41 = \frac{9}{2}k + \left(25 - \frac{1}{2}k\right) = 25 + 4k \longrightarrow$

$4k = 16 \longrightarrow k = 4 \longrightarrow C = 23$ *so*

$\boxed{T = 2t^2 + 23}$

d.) *If* $t = 4$, *then*

$T = 2(4)^2 + 23 = 55\ °F$

Example 2:

a.) $\frac{dV}{dx} = k \cdot \frac{1}{V}$

b.) $\int V\,dV = \int k\,dx \longrightarrow$

$\frac{1}{2}V^2 = kx + C$

c.) $\begin{cases} x=10, V=40 \\ x=15, V=30 \end{cases} \longrightarrow \begin{cases} 800 = 10k + C \\ 450 = 15k + C \end{cases}$

$\longrightarrow C = 800 - 10k \longrightarrow$ (SUB) $\longrightarrow$

$450 = 15k + (800 - 10k) \longrightarrow$

$-350 = 5k \longrightarrow \boxed{k = -70} \longrightarrow \boxed{C = 1500}$

$\longrightarrow \frac{1}{2}V^2 = -70x + 1500 \longrightarrow$

$V^2 = 3000 - 140x \longrightarrow$

$\boxed{V = \sqrt{3000 - 140x}}$

d.) If $x = 18$, then

$V = \sqrt{3000 - 140(18)} = \sqrt{480}$

$\approx \boxed{21.9 \text{ in.}^3}$