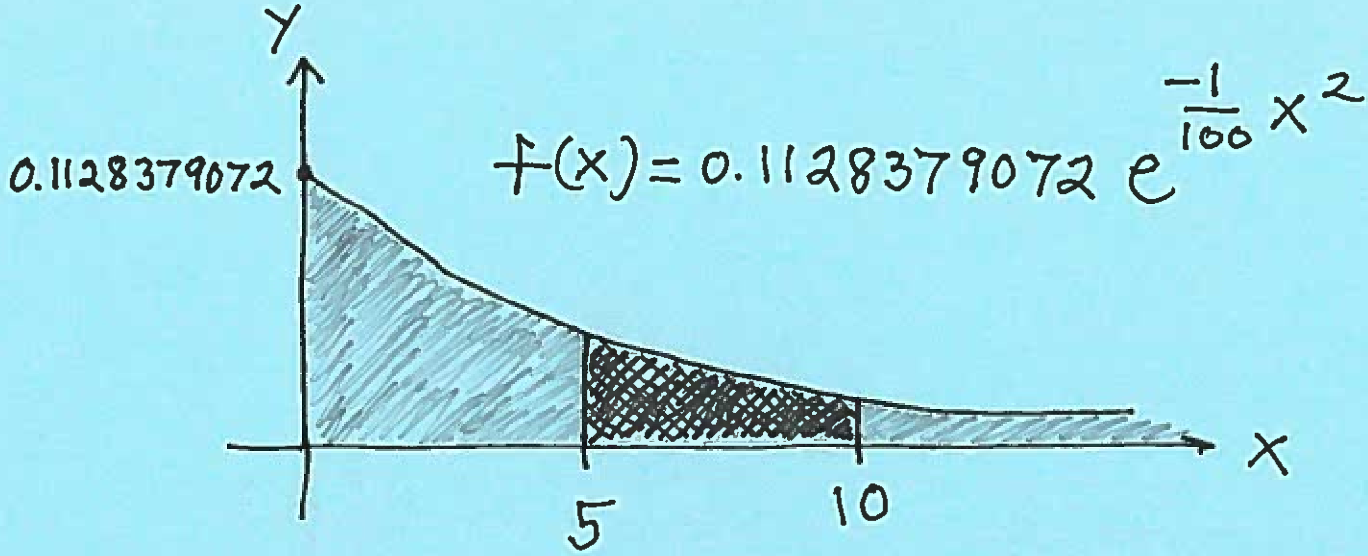
Math 16C
Kouba
Coffee and Taylor Polynomials

Example: assume that the number of cups of coffee that you drink during Final Exam Week is given by the Probability Density Function

$$f(x) = 0.1128379072\, e^{\frac{-1}{100} x^2},$$

where x is the # of cups of coffee and $0 \le x < \infty$.

Task: Use an 8th-degree Taylor Polynomial centered at $x=0$ to ESTIMATE the probability that you will drink between 5 and 10 cups of coffee during Final Exam Week.



We know that

$$e^x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \cdots$$

so that

$$e^{\frac{-1}{100}x^2} = 1 + \left(\frac{-1}{100}x^2\right) + \frac{1}{2}\left(\frac{-1}{100}x^2\right)^2 + \frac{1}{6}\left(\frac{-1}{100}x^2\right)^3 + \frac{1}{24}\left(\frac{-1}{100}x^2\right)^4 + \cdots$$

$$= 1 - \frac{1}{100}x^2 - \frac{1}{20000}x^4 - \frac{1}{6000000}x^6 + \frac{1}{2400000000}x^8 - \cdots ;$$

then the 8th-degree Taylor Polynomial is

$$P_8(x) = 1 - \frac{1}{100}x^2 - \frac{1}{20000}x^4 - \frac{1}{6000000}x^6 + \frac{1}{2400000000}x^8.$$

Now the probability of drinking between 5 and 10 cups of coffee is

$$P(5 \le X \le 10) = \int_5^{10} 0.1128379072\, e^{-\frac{1}{100}x^2}\, dx$$

$$\approx \int_5^{10} P_8(x)\, dx$$

$$= \int_5^{10} \left[1 - \frac{1}{100}x^2 - \frac{1}{20000}x^4 - \frac{1}{6000000}x^6 + \frac{1}{2400000000}x^8\right] dx$$

$$= \cdots \approx 0.3229 = 32.29\%$$

EXACT: (Using WolframAlpha)

$$P(5 \le X \le 10) = \int_5^{10} 0.1128379072\, e^{-\frac{1}{100}x^2}\, dx$$

$$\approx 0.3222 = 32.22\%$$