Math 16C
Kouba
Coffee and Taylor Polynomials

Example: Assume that the number of cups of coffee that you drink during Final Exam Week is given by the Probability Density Function

\[ f(x) = 0.1128379072 e^{-\frac{1}{100} x^2} \]

where \( x \) is the \# of cups of coffee and \( 0 \leq x < \infty \).

Task: Use an 8th-degree Taylor Polynomial centered at \( x=0 \) to ESTIMATE the probability that you will drink between 5 and 10 cups of coffee during Final Exam Week.
We know that
\[ e^x = 1 + x + \frac{1}{2!} x^2 + \frac{1}{3!} x^3 + \frac{1}{4!} x^4 + \ldots \]
so that
\[ e^{-\frac{1}{100} x^2} = 1 + \left( -\frac{1}{100} x^2 \right) + \frac{1}{2} \left( -\frac{1}{100} x^2 \right)^2 \]
\[ + \frac{1}{6} \left( -\frac{1}{100} x^2 \right)^3 + \frac{1}{24} \left( -\frac{1}{100} x^2 \right)^4 + \ldots \]
\[ = 1 - \frac{1}{100} x^2 - \frac{1}{20000} x^4 \]
\[ - \frac{1}{6000000} x^6 + \frac{1}{2400000000} x^8 - \ldots \]

then the 8th-degree Taylor polynomial is
\[ p_8(x) = 1 - \frac{1}{100} x^2 - \frac{1}{20000} x^4 \]
\[ - \frac{1}{6000000} x^6 + \frac{1}{2400000000} x^8. \]

Now the probability of drinking between 5 and 10 cups of coffee is

\[ p(5 \leq x \leq 10) = \int_5^{10} \frac{-1}{100} x^2 \]
\[ \approx \int_5^{10} p_8(x) \, dx \]
\[ = \int_5^{10} \left[ 1 - \frac{1}{100} x^2 - \frac{1}{20000} x^4 \right. \]
\[ - \frac{1}{6000000} x^6 + \frac{1}{2400000000} x^8 \left] \, dx \right. \]
\[ = \cdots \approx 0.3229 = 32.29 \% \]

**EXACT**: (Using WolframAlpha)

\[ p(5 \leq x \leq 10) = \int_5^{10} 0.1128379072 e^{-\frac{1}{100} x^2} \, dx \]
\[ \approx 0.3222 = 32.22 \% \]