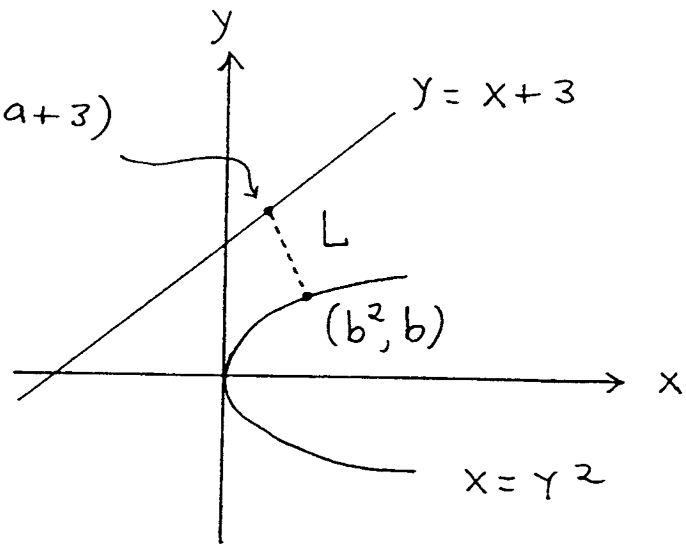
Math 16C
Kouba
Extrema Problem

Ex: Find the shortest distance between the graphs of $Y = X + 3$ and $X = Y^2$ :

Determine numbers a and b so that we minimize distance L given by




$$L = \sqrt{(a-b^2)^2 + (a+3-b)^2} \; ;$$

Find the critical point(s) for this function of two variables $\longrightarrow$

$$\left.\begin{aligned} L_a &= \tfrac{1}{2}(\text{——})^{-\frac{1}{2}} \cdot [2(a-b^2) + 2(a+3-b)] = 0 \\ L_b &= \tfrac{1}{2}(\text{——})^{-\frac{1}{2}} \cdot [2(a-b^2)(-2b) + 2(a+3-b)(-1)] = 0 \end{aligned}\right\} \rightarrow$$

$$(*) \quad \left.\begin{aligned} (a-b^2) + (a+3-b) &= 0 \\ (a-b^2)(-2b) - (a+3-b) &= 0 \end{aligned}\right\} \text{add equations} \longrightarrow$$

$$(a-b^2)(1-2b) = 0 \rightarrow \quad a = b^2 \text{ or } b = \tfrac{1}{2} \; ;$$

if $\underline{\underline{a = b^2}}$ then equation (*) becomes

$$(b^2 - b^2) + (b^2 + 3 - b) = 0 \quad \longrightarrow$$

(quadratic formula) $\quad b = \dfrac{1 \pm \sqrt{1-12}}{2}$

(These are complex numbers, so $a = b^2$ does not create a critical point.) ;

if $\underline{\underline{b = \frac{1}{2}}}$ then equation (*) becomes

$$\left(a - \tfrac{1}{4}\right) + \left(a + 3 - \tfrac{1}{2}\right) = 0 \quad \longrightarrow$$

$$2a = \frac{-9}{4} \quad \longrightarrow \quad \underline{\underline{a = \frac{-9}{8}}} \quad ;$$

$a = \frac{-9}{8}$ determines the pt. $\left(-\frac{9}{8}, \frac{15}{8}\right)$ and
$b = \frac{1}{2}$ determines the pt. $\left(\frac{1}{4}, \frac{1}{2}\right)$ with

a minimum distance of

$$L = \sqrt{\left(\tfrac{1}{4} + \tfrac{9}{8}\right)^2 + \left(\tfrac{1}{2} - \tfrac{15}{8}\right)^2} \approx 1.945$$