Ex: Find the shortest distance between the graphs of $Y = X+3$ and $X = Y^2$:

Determine numbers $a$ and $b$ so that we minimize distance $L$ given by

$$L = \sqrt{(a-b^2)^2 + (a+3-b)^2}$$

Find the critical point(s) for this function of two variables $\rightarrow$

$$L_a = \frac{1}{2} (a-b^2)^2 \cdot [2(a-b^2) + 2(a+3-b)] = 0$$
$$L_b = \frac{1}{2} (a+3-b)^2 \cdot [2(a-b^2)(-2b) + 2(a+3-b)(1)] = 0$$

$$(a-b^2) + (a+3-b) = 0 \quad \text{add}$$
$$(a-b^2)(-2b) - (a+3-b) = 0 \quad \text{equations} \rightarrow$$

$$(a-b^2)(1-2b) = 0 \rightarrow a = b^2 \quad \text{or} \quad b = \frac{1}{2} \; ;$$
if $a = b^2$ then equation (*) becomes

$$(b^2 - b^2) + (b^2 + 3 - b) = 0 \quad \rightarrow \quad (\text{quadratic formula}) \quad b = \frac{1 \pm \sqrt{1-12}}{2}$$

(These are complex numbers, so $a = b^2$ does not create a critical point.)

if $b = \frac{1}{2}$ then equation (*) becomes

$$(a - \frac{1}{4}) + (a + 3 - \frac{1}{2}) = 0 \quad \rightarrow \quad 2a = \frac{-9}{4} \quad \rightarrow \quad a = \frac{-9}{8}$$

$a = \frac{-9}{8}$ determines the pt. $\left( -\frac{9}{8}, \frac{15}{8} \right)$ and

$b = \frac{1}{2}$ determines the pt. $\left( \frac{1}{4}, \frac{1}{2} \right)$ with a minimum distance of

$$L = \sqrt{\left( \frac{1}{4} + \frac{9}{8} \right)^2 + \left( \frac{1}{2} - \frac{15}{8} \right)^2} \approx 1.945$$