Math 16C
Kouba
Worksheet 8

1.) Evaluate the following double integrals. Realize that in some cases you must switch the order of integration before you compute the antiderivatives.

a.) $\int_0^{\pi/4} \int_0^{\pi} \sec^2(x/3 + y/4)\ dy\ dx$

b.) $\int_0^{2\pi} \int_0^{3\pi} \cos(x/3 - y/2)\ dy\ dx$

c.) $\int_0^{\pi/2} \int_0^{\pi/6} \sin^2(x - y)\ dy\ dx$

d.) $\int_1^{5} \int_0^{3/\sqrt{x}} 2xy \cdot \tan^2(xy^2)\ dy\ dx$

e.) $\int_0^{4} \int_{\sqrt{y}}^{2} y \cdot \cos(x^5)\ dx\ dy$

f.) $\int_0^{1} \int_{\ln(x+1)}^{\ln 2} \frac{2x}{e^{2y} - 2e^y + 1}\ dy\ dx$

2.) Sketch the volume of the solid which is represented by each of the following double integrals. You need not compute the value of the double integral.

a.) $\int_0^{3} \int_0^{3-x} 1/3(12 - x - 2y)\ dy\ dx$

b.) $\int_0^{2} \int_0^{y/2} (x^2 + y^2)\ dx\ dy$

Worksheet 8

1.) a.) $\int_0^{\pi/4}\int_0^{\pi} \sec^2\left(\frac{x}{3}+\frac{y}{4}\right) dy\, dx = \int_0^{\frac{\pi}{4}} 4\tan\left(\frac{x}{3}+\frac{y}{4}\right)\Big|_{y=0}^{y=\pi} dx$

$= \int_0^{\frac{\pi}{4}} \left[4\tan\left(\frac{x}{3}+\frac{\pi}{4}\right) - 4\tan\left(\frac{x}{3}\right)\right] dx$

$= 12\ln\left|\sec\left(\frac{x}{3}+\frac{\pi}{4}\right)\right| - 12\ln\left|\sec\left(\frac{x}{3}\right)\right| \Big|_0^{\frac{\pi}{4}}$

$= 12\ln\left|\sec\left(\frac{\pi}{12}+\frac{\pi}{4}\right)\right| - 12\ln\left|\sec\left(\frac{\pi}{12}\right)\right|$
$\quad - \left(12\ln\left|\sec\left(\frac{\pi}{4}\right)\right| - 12\ln\left|\sec 0\right|\right)$

$= 12\ln(2) - 12\ln\left|\sec\left(\frac{\pi}{12}\right)\right| - 12\ln(\sqrt{2}) - \overbrace{12\ln(1)}^{0}$

$= 12\ln\left(\frac{2}{\sqrt{2}}\right) - 12\ln\left|\sec\left(\frac{\pi}{12}\right)\right| = 12\ln\sqrt{2} - 12\ln\left|\sec\left(\frac{\pi}{12}\right)\right|$

b.) $\int_0^{2\pi}\int_0^{3\pi} \cos\left(\frac{x}{3}-\frac{y}{2}\right) dy\, dx$

$= \int_0^{2\pi} -2\sin\left(\frac{x}{3}-\frac{y}{2}\right)\Big|_{y=0}^{y=3\pi} dx$

$= \int_0^{2\pi} \left[-2\sin\left(\frac{x}{3}-\frac{3\pi}{2}\right) - {}^{-}2\sin\left(\frac{x}{3}\right)\right] dx$

$= 6\cos\left(\frac{x}{3}-\frac{3\pi}{2}\right) - 6\cos\left(\frac{x}{3}\right)\Big|_0^{2\pi}$

$= \left[6\cos\left(\frac{2\pi}{3}-\frac{3\pi}{2}\right) - 6\cos\left(\frac{2\pi}{3}\right)\right] - \left[6\cos\left(-\frac{3\pi}{2}\right) - 6\cos(0)\right]$

$= 6\cdot\left(-\frac{\sqrt{3}}{2}\right) - 6\left(\frac{-1}{2}\right) - 6(0) + 6(1) = 9 - 3\sqrt{3}$

$\cos 2\theta = 1 - 2\sin^2\theta$
(Use trig identity)

c.) $\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{6}} \sin^2(x-y)\, dy\, dx$

$= \int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{6}} \frac{1}{2}\left[1 - \cos 2(x-y)\right] dy\, dx$

$= \int_0^{\frac{\pi}{2}} \frac{1}{2}\left[y + \frac{1}{2}\sin(2x-2y)\right]\Big|_{y=0}^{y=\frac{\pi}{6}} dx$

$$= \int_0^{\frac{\pi}{2}} \frac{1}{2}\left[\frac{\pi}{6} + \frac{1}{2}\sin\left(2x - \frac{\pi}{3}\right)\right] - \frac{1}{2}\left[0 + \frac{1}{2}\sin(2x)\right] \, dx$$

$$= \int_0^{\frac{\pi}{2}} \left[\frac{\pi}{12} + \frac{1}{4}\sin\left(2x - \frac{\pi}{3}\right) - \frac{1}{4}\sin(2x)\right] dx$$

$$= \frac{\pi}{12}x - \frac{1}{8}\cos\left(2x - \frac{\pi}{3}\right) + \frac{1}{8}\cos(2x) \Big|_0^{\frac{\pi}{2}}$$

$$= \frac{\pi^2}{24} - \frac{1}{8}\cos\left(\frac{2}{3}\pi\right) + \frac{1}{8}\cos(\pi) - \left(0 - \frac{1}{8}\cos\left(-\frac{\pi}{3}\right) + \frac{1}{8}\cos(0)\right)$$

$$= \frac{\pi^2}{24} - \frac{1}{8}\left(\frac{-1}{2}\right) + \frac{1}{8}(-1) + \frac{1}{8}\left(\frac{1}{2}\right) - \frac{1}{8}(1)$$

$$= \frac{\pi^2}{24} - \frac{1}{8}$$

$1 + \tan^2\theta = \sec^2\theta$

d.) $\int_1^5 \int_0^{3/\sqrt{x}} 2xy \tan^2(xy^2) \, dy \, dx$ (Use trig identity)

$$= \int_1^5 \int_0^{3/\sqrt{x}} 2xy\left[\sec^2(xy^2) - 1\right] dy \, dx$$

$$= \int_1^5 \int_0^{3/\sqrt{x}} \left[2xy\sec^2(xy^2) - 2xy\right] dy \, dx$$

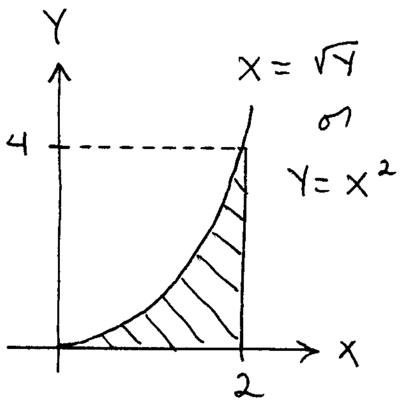
$$= \int_1^5 \left[\tan(xy^2) - xy^2\right] \Big|_{y=0}^{y=3/\sqrt{x}} dx$$

$$= \int_1^5 \left[\tan(9) - 9\right] dx = (\tan(9) - 9)x \Big|_1^5$$

$$= 4(\tan(9) - 9)$$

e.) $\int_0^4 \int_{\sqrt{y}}^2 y\cos(x^5) \, dx \, dy$

(SWITCH ORDER OF INTEGRATION)



$$= \int_0^2 \int_0^{x^2} y \cos(x^5)\, dy\, dx$$

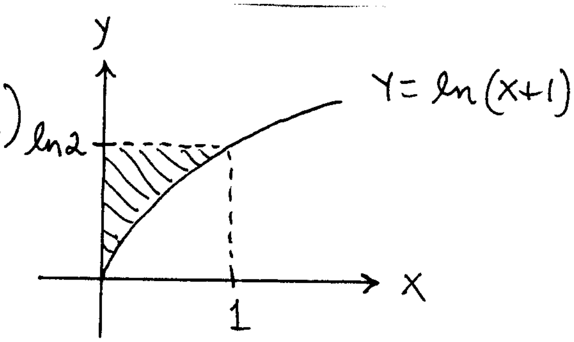
$$= \int_0^2 \frac{y^2}{2} \cos(x^5) \Big|_{y=0}^{y=x^2} dx$$

$$= \int_0^2 \frac{1}{2} x^4 \cos(x^5)\, dx$$

$$= \frac{1}{10} \sin(x^5) \Big|_0^2 = \frac{1}{10}\sin(32) - \frac{1}{10}\overset{0}{\cancel{\sin(0)}}$$

$$= \frac{1}{10}\sin(32)$$

f.)



$$\int_0^1 \int_{\ln(x+1)}^{\ln 2} \frac{2x}{e^{2y} - 2e^{y} + 1}\, dy\, dx$$

(SWITCH ORDER !!)

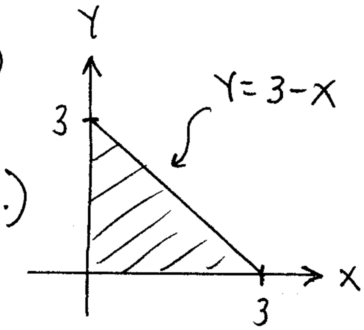
$$= \int_0^{\ln 2} \int_0^{e^y - 1} \frac{2x}{(e^y-1)^2}\, dx\, dy = \int_0^{\ln 2} \frac{x^2}{(e^y-1)^2} \Big|_{x=0}^{x=e^y-1} dy$$

$$= \int_0^{\ln 2} \frac{(e^y-1)^2}{(e^y-1)^2}\, dy = \int_0^{\ln 2} 1\, dy$$

$$= y \Big|_0^{\ln 2} = \ln 2$$

2.)

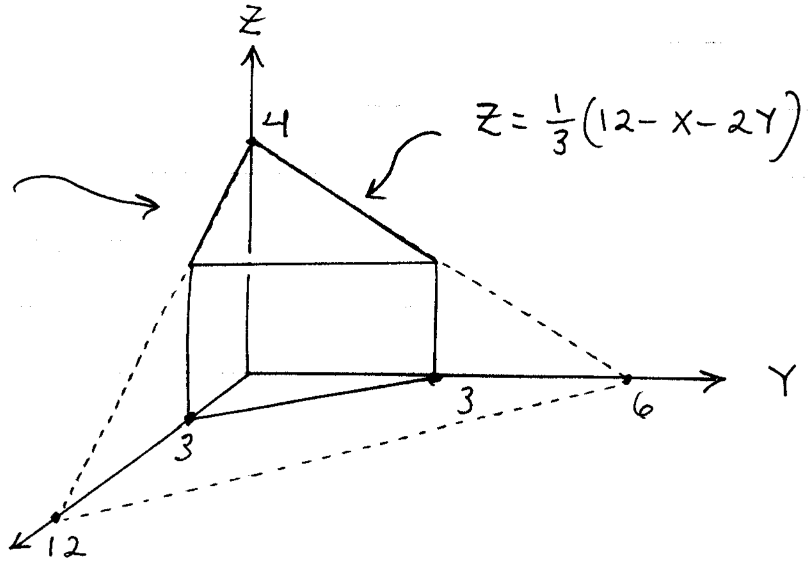
a.)



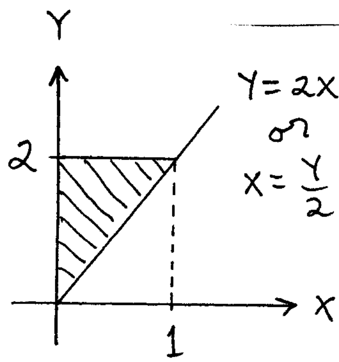
$Z = \frac{1}{3}(12 - X - 2Y) \rightarrow$

$X + 2Y + 3Z = 12$ so surface is a plane crossing the axes at $X = 12$, $Y = 6$, and $Z = 4$:

4-sided solid



b.)



$Z = X^2 + Y^2$ is a paraboloid passing through the origin :