Section 10.1

9.) $\lim_{n\to\infty} \frac{5}{n} = "\frac{5}{\infty}" = 0$

10.) $\lim_{n\to\infty} \frac{n}{2} = "\frac{\infty}{2}" = \infty$ (diverges)

11.) $\lim_{n\to\infty} \frac{n+1}{n} = \lim_{n\to\infty} \left(1+\frac{1}{n}\right) = 1+0 = 1$

12.) $\lim_{n\to\infty} \frac{1}{n^{3/2}} = "\frac{1}{\infty}" = 0$

13.) $\lim_{n\to\infty} \frac{n^2+3n-4}{2n^2+n-3} = \lim_{n\to\infty} \frac{1+\frac{3}{n}-\frac{4}{n^2}}{2+\frac{1}{n}-\frac{3}{n^2}}$

$= \frac{1+0-0}{2+0-0} = \frac{1}{2}$

14.) $\lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n+1}} = \lim_{n\to\infty} \sqrt{\frac{n}{n+1}} = \lim_{n\to\infty} \sqrt{\frac{1}{1+1/n}}$

$= \sqrt{\frac{1}{1+0}} = \sqrt{1} = 1$

15.) $\lim_{n\to\infty} \frac{n^2-25}{n+5} = \lim_{n\to\infty} \frac{(n-5)\cancel{(n+5)}}{\cancel{n+5}}$

$= \infty$ (diverges)

16.) $\lim_{n\to\infty} \frac{n+2}{n^2+1} = \lim_{n\to\infty} \frac{\frac{1}{n}+\frac{2}{n^2}}{1+\frac{1}{n^2}} = \frac{0+0}{1+0} = 0$

17.) $0 \le \frac{1+(-1)^n}{n} \le \frac{2}{n}$ and $\lim_{n\to\infty} \frac{2}{n} = 0$

so $\lim_{n\to\infty} \frac{1+(-1)^n}{n} = 0$ by "squeeze"

18.) $a_n = 1+(-1)^n$: 0, 2, 0, 2, 0, 2, ...

position : 1 2 3 4 5 6 ...

$\lim_{n\to\infty} (1+(-1)^n)$ DNE by oscillation

19.) $\lim_{n\to\infty} \frac{n!}{n} = \lim_{n\to\infty} \frac{\cancel{n}(n-1)(n-2)\cdots(3)(2)(1)}{\cancel{n}}$

$= \infty$ (diverges)

20.) $\lim_{n\to\infty} \frac{n!}{(n+1)!} = \lim_{n\to\infty} \frac{\cancel{n}\cancel{(n-1)}\cancel{(n-2)}\cdots\cancel{(3)}\cancel{(2)}\cancel{(1)}}{(n+1)\cancel{n}\cancel{(n-1)}\cancel{(n-2)}\cdots\cancel{(3)}\cancel{(2)}\cancel{(1)}}$

$= \lim_{n\to\infty} \frac{1}{n+1} = \text{"}\frac{1}{\infty}\text{"} = 0$

21.) $\lim_{n\to\infty} \left(3 - \frac{1}{2^n}\right) = 3 - \text{"}\frac{1}{\infty}\text{"} = 3 - 0 = 3$

22.) $\lim_{n\to\infty} \frac{n}{\sqrt{n^2+1}} = \lim_{n\to\infty} \frac{n}{\sqrt{n^2(1+\frac{1}{n^2})}}$

$= \lim_{n\to\infty} \frac{\cancel{n}}{\cancel{n}\sqrt{1+1/n^2}} = \lim_{n\to\infty} \frac{1}{\sqrt{1+1/n^2}} = \frac{1}{\sqrt{1+0}} = 1$

23.) $\lim_{n\to\infty} \frac{3^n}{4^n} = \lim_{n\to\infty} \left(\frac{3}{4}\right)^n = 0$ since $-1 < 3/4 < 1$

24.) $\lim_{n\to\infty} (0.5)^n = 0$ since $-1 < 0.5 < 1$

25.) $\lim_{n\to\infty} \frac{(n+1)!}{n!} = \lim_{n\to\infty} \frac{(n+1)\cancel{n}\cancel{(n-1)}\cancel{(n-2)}\cdots\cancel{(3)}\cancel{(2)}\cancel{(1)}}{\cancel{n}\cancel{(n-1)}\cancel{(n-2)}\cdots\cancel{(3)}\cancel{(2)}\cancel{(1)}}$

$= \lim_{n\to\infty} (n+1) = \infty$ (diverges)

26.) $\lim_{n\to\infty} \frac{(n-2)!}{n!} = \lim_{n\to\infty} \frac{\cancel{(n-2)}\cancel{(n-3)}\cdots\cancel{(3)}\cancel{(2)}\cancel{(1)}}{n(n-1)\cancel{(n-2)}\cancel{(n-3)}\cdots\cancel{(3)}\cancel{(2)}\cancel{(1)}}$

$$= \lim_{n\to\infty} \frac{1}{n(n-1)} = \text{"}\frac{1}{\infty}\text{"} = 0$$

27.) $\lim_{n\to\infty} \frac{n}{n+1} = \lim_{n\to\infty} \frac{1}{1+\frac{1}{n}} = \frac{1}{1+0} = 1$

so $\lim_{n\to\infty} (-1)^n \cdot \frac{n}{n+1}$ diverges by oscillation

28.) $\lim_{n\to\infty} \left(\frac{n-1}{n} - \frac{n}{n-1}\right)$

$$= \lim_{n\to\infty} \frac{(n-1)^2 - n^2}{n(n-1)} = \lim_{n\to\infty} \frac{\cancel{n^2} - 2n + 1 - \cancel{n^2}}{n^2 - n}$$

$$= \lim_{n\to\infty} \frac{-2 - \frac{1}{n}}{n-1} = \text{"}\frac{-2-0}{\infty}\text{"} = 0$$

Section 10.2

13.) $\sum_{n=0}^{\infty} 2\left(\frac{3}{4}\right)^n$ converges since $r = 3/4$ and $-1 < r < 1$.

14.) $\sum_{n=0}^{\infty} 2\left(\frac{-1}{2}\right)^n$ converges since $r = -1/2$ and $-1 < r < 1$.

15.) $\sum_{n=0}^{\infty} (0.9)^n$ converges since $r = 0.9$ and $-1 < r < 1$.

16.) $\sum_{n=0}^{\infty} (-0.6)^n$ converges since $r = -0.6$ and $-1 < r < 1$.

18.) $$\sum_{n=0}^{\infty} 2\left(\frac{2}{3}\right)^n = 2\sum_{n=0}^{\infty}\left(\frac{2}{3}\right)^n = 2\left(1+\left(\frac{2}{3}\right)+\left(\frac{2}{3}\right)^2+\left(\frac{2}{3}\right)^3+\cdots\right)$$
$$= 2\cdot\frac{1}{1-\left(\frac{2}{3}\right)} = 2\cdot 3 = 6$$

19.) $$\sum_{n=0}^{\infty}\left(\frac{-1}{2}\right)^n = 1+\left(\frac{-1}{2}\right)+\left(\frac{-1}{2}\right)^2+\left(\frac{-1}{2}\right)^3+\cdots = \frac{1}{1-\left(\frac{-1}{2}\right)} = \frac{2}{3}$$

24.) $$8+6+\frac{9}{2}+\frac{27}{8}+\cdots = 8\left[1+\frac{3}{4}+\frac{9}{16}+\frac{27}{64}+\cdots\right]$$
$$= 8\left[1+\left(\frac{3}{4}\right)+\left(\frac{3}{4}\right)^2+\left(\frac{3}{4}\right)^3+\cdots\right] = 8\cdot\frac{1}{1-\left(\frac{3}{4}\right)} = 8\cdot 4 = 32$$

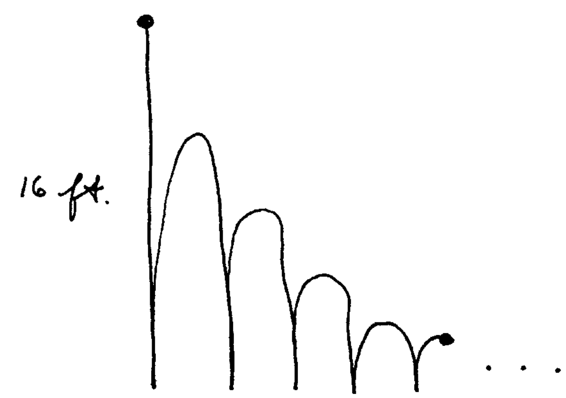
26.) $4 - 2 + 1 - \frac{1}{2} + \cdots = 4\left[1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \cdots\right]$

$= 4\left[1 + \left(\frac{-1}{2}\right) + \left(\frac{-1}{2}\right)^2 + \left(\frac{-1}{2}\right)^3 + \cdots\right] = 4 \cdot \frac{1}{1-\left(\frac{-1}{2}\right)} = 4 \cdot \frac{2}{3} = \frac{8}{3}$

28.) $\sum_{n=0}^{\infty}\left[(0.7)^n + (0.9)^n\right] = \sum_{n=0}^{\infty}(0.7)^n + \sum_{n=0}^{\infty}(0.9)^n$

$= \frac{1}{1-(0.7)} + \frac{1}{1-(0.9)} = \frac{10}{3} + \frac{10}{1} = \frac{40}{3}$

47.)



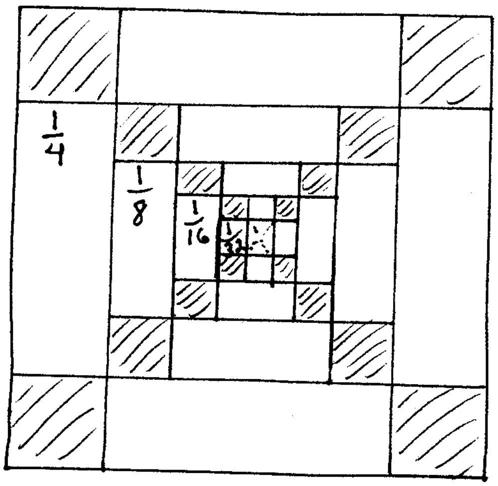
Total distance traveled is

$T = 16 + (0.64)(16)(2) + (0.64)^2(16)(2) + (0.64)^3(16)(2) + \cdots$

$= 16 + (0.64)(16)(2)\left[1 + (0.64) + (0.64)^2 + (0.64)^3 + \cdots\right]$

$= 16 + (20.48)\frac{1}{1-(0.64)} \approx 72.89$ ft.

52.)



Assume large square is 1 × 1 then total shaded area is

$$T = 4 \cdot \left(\tfrac{1}{4}\right)^2 + 4\left(\tfrac{1}{8}\right)^2 + 4\left(\tfrac{1}{16}\right)^2 + 4\left(\tfrac{1}{32}\right)^2 + \cdots$$

$$= \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \frac{1}{256} + \cdots$$

$$= \frac{1}{4}\left[1 + \left(\tfrac{1}{4}\right) + \left(\tfrac{1}{4}\right)^2 + \left(\tfrac{1}{4}\right)^3 + \cdots\right]$$

$$= \frac{1}{4} \cdot \frac{1}{1-\left(\frac{1}{4}\right)} = \frac{1}{4} \cdot \frac{4}{3} = \frac{1}{3}$$ of total area.

51.) Total \$ spent is (in millions)

$$100 + (.75)100 + (.75)^2 100 + (.75)^3 100 + \cdots$$

$$= 100\left[1 + (.75) + (.75)^2 + (.25)^3 + \cdots\right]$$

$$= 100 \cdot \frac{1}{1-(.75)} = 400$$ (million \$)