Section 10.2 (cont'd.)

6.) $\lim_{n\to\infty} \frac{n}{2n+3} = \lim_{n\to\infty} \frac{1}{2+\frac{3}{n}} = \frac{1}{2+0} = \frac{1}{2} \neq 0$

so $\sum_{n=1}^{\infty} \frac{n}{2n+3}$ diverges.

7.) $\lim_{n\to\infty} \frac{n^2}{n^2+1} = \lim_{n\to\infty} \frac{1}{1+\frac{1}{n^2}} = \frac{1}{1+0} = 1 \neq 0$

so $\sum_{n=1}^{\infty} \frac{n^2}{n^2+1}$ diverges.

8.) $\lim_{n\to\infty} \frac{n}{\sqrt{n^2+1}} = \lim_{n\to\infty} \sqrt{\frac{n^2}{n^2+1}} = \lim_{n\to\infty} \sqrt{\frac{1}{1+\frac{1}{n^2}}} = \sqrt{1} = 1 \neq 0$

so $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^2+1}}$ diverges.

9.) $\lim_{n\to\infty} 3\left(\frac{3}{2}\right)^n = \infty$ (since $\frac{3}{2} > 1$) $\neq 0$ so

$\sum_{n=0}^{\infty} 3\left(\frac{3}{2}\right)^n$ diverges

12.) $\lim_{n\to\infty} 2(1.03)^n = +\infty$ (since $1.03 > 1$) so $\lim_{n\to\infty} 2(-1.03)^n$ DNE (oscillation) so $\lim_{n\to\infty} 2(-1.03)^n \neq 0$ and $\sum_{n=0}^{\infty} 2(-1.03)^n$ diverges.

31.) $\lim_{n\to\infty} \frac{n+10}{10n+1} = \lim_{n\to\infty} \frac{1+\frac{10}{n}}{10+\frac{1}{n}} = \frac{1+0}{10+0} = \frac{1}{10} \neq 0$

so series $\sum_{n=1}^{\infty} \frac{n+10}{10n+1}$ diverges by nth-term test

32.) $\sum_{n=0}^{\infty} \frac{4}{2^n} = 4 \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n = 4\left(1+\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2+\cdots\right)$

$= 4 \cdot \frac{1}{1-\frac{1}{2}} = 4 \cdot 2 = 8$ converges by geom. series test

33.) $\lim_{n\to\infty} \frac{n+1}{n} = \lim_{n\to\infty} \left(1+\frac{1}{n}\right) = 1+0 = 1 \neq 0$

so series $\sum_{n=1}^{\infty} \frac{n+1}{n}$ diverges by nth-term test

34.) $\lim_{n\to\infty} \frac{n+1}{2n-1} = \lim_{n\to\infty} \frac{1+\frac{1}{n}}{2-\frac{1}{n}} = \frac{1}{2} \neq 0$

so series $\sum_{n=1}^{\infty} \frac{n+1}{2n-1}$ diverges by nth-term test

35.) $\lim_{n\to\infty} \frac{3n-1}{2n+1} = \lim_{n\to\infty} \frac{3-\frac{1}{n}}{2+\frac{1}{n}} = \frac{3}{2} \neq 0$

so series $\sum_{n=1}^{\infty} \frac{3n-1}{2n+1}$ diverges by nth-term test

36.) $\sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n = 1+\left(\frac{1}{4}\right)+\left(\frac{1}{4}\right)^2+\cdots$

$= \frac{1}{1-\frac{1}{4}} = \frac{4}{3}$ converges by geom. series test

37.) $\sum_{n=0}^{\infty} (1.075)^n$ diverges since $1.075 \geq 1$ by geom. series test

38.) $\sum_{n=1}^{\infty} \frac{2^n}{100} = \frac{1}{100} \sum_{n=1}^{\infty} 2^n$ diverges

since $2 \geq 1$ by geom. series test

39.) $\sum_{n=0}^{\infty} \frac{3}{4^n} = 3 \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n$ converges

by geometric series test since $-1 < \frac{1}{4} < 1$

40.) $\sum_{n=0}^{\infty} n!$ diverges since

$\lim_{n\to\infty} n! \neq 0$ by nth-term test

45.)

a.)

after # of years	units in use
1	$(0.9)\,8000$
2	$(0.9)\,8000 + (0.9)^2\,8000$
3	$(0.9)\,8000 + (0.9)^2\,8000 + (0.9)^3\,8000$
$\vdots$	$\vdots$
n	$(0.9)8000 + (0.9)^2 8000 + (0.9)^3 8000 + \cdots + (0.9)^n 8000$

$$= (0.9)\,8000\left[1 + (0.9) + (0.9)^2 + (0.9)^3 + \cdots + (0.9)^{n-1}\right]$$

$$= (0.9)\,8000 \cdot \frac{1-(0.9)^n}{1-(0.9)} = 72{,}000\left(1-(0.9)^n\right) \text{ units}$$

b.) $\lim_{n\to\infty} 72{,}000\left(1-(0.9)^n\right) = 72{,}000$ units

49.) $A = 100\left(1+\frac{0.10}{12}\right) + 100\left(1+\frac{0.10}{12}\right)^2 + \cdots + 100\left(1+\frac{0.10}{12}\right)^{60}$

$= 100\left(1+\frac{0.10}{12}\right) \cdot \left[1 + \left(1+\frac{0.10}{12}\right) + \left(1+\frac{0.10}{12}\right)^2 + \cdots + \left(1+\frac{0.10}{12}\right)^{59}\right]$

$= 100\left(1+\frac{0.10}{12}\right) \cdot \dfrac{1-\left(1+\frac{0.10}{12}\right)^{59+1}}{1-\left(1+\frac{0.10}{12}\right)} \approx \$7808.24$

50.) $A = P\left(1+\frac{r}{12}\right) + P\left(1+\frac{r}{12}\right)^2 + \cdots + P\left(1+\frac{r}{12}\right)^N$

$= P\left(1+\frac{r}{12}\right)\left[1 + \left(1+\frac{r}{12}\right) + \left(1+\frac{r}{12}\right)^2 + \cdots + \left(1+\frac{r}{12}\right)^{N-1}\right]$

$= P\left(1+\frac{r}{12}\right) \cdot \dfrac{1-\left(1+\frac{r}{12}\right)^N}{1-\left(1+\frac{r}{12}\right)} = P\left(1+\frac{r}{12}\right) \cdot \frac{-12}{r}\left[1-\left(1+\frac{r}{12}\right)^N\right]$

$= P\left(\frac{12}{r}+1\right)\left[\left(1+\frac{r}{12}\right)^N - 1\right]$.

53.)

day	\$ (in cents)
1	1
2	$2 = 2^1$
3	$4 = 2^2$
4	$8 = 2^3$
$\vdots$	$\vdots$
n	2^{n-1}

so total \$ for 20 days

is $T = 1 + 2 + 2^2 + 2^3 + 2^4 + \cdots + 2^{19}$

$= \dfrac{1-2^{20}}{1-2} = 1{,}048{,}575\,¢ = \$10{,}485.75$

Section 10.3

7.) $\sum_{n=1}^{\infty} \frac{1}{n^3}$ converges since $p=3>1$.

8.) $\sum_{n=1}^{\infty} \frac{1}{n^{1/3}}$ diverges since $p=\frac{1}{3} \le 1$.

9.) $\sum_{n=1}^{\infty} \frac{1}{n^{1/3}}$ diverges since $p=\frac{1}{3} \le 1$.

10.) $\sum_{n=1}^{\infty} \frac{1}{n^{4/3}}$ converges since $p=\frac{4}{3}>1$.

11.) $\sum_{n=1}^{\infty} \frac{1}{n^{1.03}}$ converges since $p=1.03>1$.

12.) $\sum_{n=1}^{\infty} \frac{1}{n^{\pi}}$ converges since $p=\pi>1$.

13.) $1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{4}}+\cdots = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$
diverges since $p=1/2 \le 1$.

14.) $1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\cdots = \sum_{n=1}^{\infty} \frac{1}{n^2}$ converges
since $p=2>1$.

15.) $1+\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}+\frac{1}{4\sqrt{4}}+\cdots$
$= 1+\frac{1}{2^{3/2}}+\frac{1}{3^{3/2}}+\frac{1}{4^{3/2}}+\cdots = \sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$
converges since $p=3/2>1$.

16.) $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots = \sum_{n=1}^{\infty} \frac{1}{n} = \sum_{n=1}^{\infty} \frac{1}{n^1}$
diverges since $p=1 \le 1$.

32.) $S = \underbrace{\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4}}_{\text{partial sum } S_4 \approx 1.078751929} + \underbrace{\frac{1}{5^4} + \frac{1}{6^4} + \cdots}_{\text{error } R_4}$;

$$R_4 < \frac{1}{(p-1)N^{p-1}} = \frac{1}{(4-1)\cdot 4^{4-1}} \approx 0.005208333$$

so $1.078751929 < S < 1.078751929 + 0.005208333$

$\rightarrow$ $1.078751929 < S < 1.083960262$

33.) $S = \underbrace{\frac{1}{1^{3/2}} + \frac{1}{2^{3/2}} + \cdots + \frac{1}{10^{3/2}}}_{\text{partial sum } S_{10} \approx 1.995336493} + \underbrace{\frac{1}{11^{3/2}} + \frac{1}{12^{3/2}} + \cdots}_{\text{error } R_{10}}$;

$$R_{10} < \frac{1}{(p-1)N^{p-1}} = \frac{1}{(\frac{3}{2}-1)10^{3/2-1}} \approx 0.632455532$$

so $1.995336493 < S < 1.995336493 + 0.632455532$

$\rightarrow$ $1.995336493 < S < 2.627792025$